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A QUADRATIC-QUARTIC-QUINTIC (QQQ) PERTURBATION ON HARMONIC OSCILLATOR: HAMILTONIAN MATRIX REPRESENTATION

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Abstract

This paper discusses a one-dimensional harmonic oscillator system subjected to simultaneous quadratic-quartic-quintic perturbation. The main objective of this paper is to calculate the matrix representation of its Hamiltonian. To keep more generally, these three perturbation terms use different small parameters. The method used in this paper is the standard algebraic method using Dirac notation with the bases that have also been shown. The results of the analysis that has been carried out, it is obtained in the form of the Hamiltonian matrix representation of this system which contains the three different small parameters. If the three parameters are chosen to be zero (without perturbation), the matrix form will be reduced to a standard harmonic oscillator matrix.

Keywords: *QQQ perturbation; harmonic oscillator, Hamiltonian matrix representation.*

INTRODUCTION

Our universe in general consists of an extraordinary number of physical phenomena ranging from subatomic structures to large scales such as galaxies, supercluster galaxies, Laniakea, cosmic networks, and so on. Due to the immense size of our universe, this of course makes it very complex and complicated to explain. Scientists have long been trying to explain and solve existing phenomena in order to understand the large-scale structure of this universe.

From its immense complexity, the universe always "gives" the beauty that is in it. This beauty, both in terms of visuals, and from the side of mathematical abstraction, of course, comes from certain physical interpretations. This makes many scientists increasingly challenged to find out the phenomena and physical entities that underlie them. However, one thing that is quite interesting is that from the complexity of our universe, there is a physical system that is found in very many physical domains, namely the harmonic oscillator. This system appears in almost all physical dynamics models, from small-scale [1–5]to large-scale structures[6,7]. For example, in quantum phenomena[1]-[2], quantum field theory[8–10], even up to the cosmological scale[6,7,11].This system has additionally been concentrated in mathematics both differential-integrally and topologically [12–14]. The thorough investigation of physics and mathematics for more than 100 years has affirmed it as one of the most important themes throughout the entire existence of physics. Nonetheless, then again, the way that this topic is as yet being considered implies that there is still a ton to investigate from this system.

It was previously stated that the study of the harmonic oscillator exists in various domains of physics, and of course, including quantum mechanics. There are various types of harmonic oscillators, namely pure harmonic oscillators that are time-independent, time-dependent, and various other forms of study. Nowadays, the study of the harmonic oscillator is no longer in the form of a pure harmonic oscillator but has undergone many modifications based on each of the physical phenomena studied[8,15,16]. In general, the modification of this system is in the form of the presence of certain perturbation terms, from first order to higher orders.

To analyze the harmonic oscillator system, there are various methods used, and one of the most frequently used mathematical methods is the algebraic approach method, such as the use of matrix mechanics and Dirac notation. The matrix representation for the states and operators for this system is of great interest because the complex formulations of the differential-integrals form as well as certain special functions such as the spherical harmonic function (which contains the Association's Legendre function) or the Laguerre function, can be represented in certain abstract notation with simpler mathematical analysis.

This paper presents an algebraic analysis of a harmonic oscillator with simultaneous perturbation of the quadratic, quartic, and quintic terms. The focus of this study is to find a matrix representation form for its Hamiltonian operator. In this analysis, three different small parameters are used for the three perturbation terms in order to make the system more general.

HARMONIC OSCILLATOR

A pure Harmonic Oscillator without perturbation has the following Hamiltonian form[1–3]

$$\widehat{H}_0 = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2 \quad (1)$$

where $\hat{p} \equiv \frac{\hbar}{i}\left(\frac{d}{dx}\right)$ is the momentum operator; $\hat{x}$ is a position operator; ω is the angular frequency of the classical oscillator, and m is the mass of the particle. The study for this paper is conducted algebraically using Dirac notation. To analyze algebraically, let equation (1) be written in the form

$$\widehat{H}_0 = \hat{A}^2 + \hat{B}^2 \quad (2)$$

where $\hat{A}^2 = \frac{1}{2} m\omega^2 \hat{x}^2$; and $\hat{B}^2 = \frac{\hat{p}^2}{2m}$.

Since $\widehat{A}$ and $\widehat{B}$ do not commute, then equation (2) can be written as

$$\widehat{H} = (\hat{A} + i\hat{B})(\hat{A} - i\hat{B}) - i[\hat{B}, \hat{A}]. \quad (3)$$

where $[\hat{B}, \hat{A}] = -\frac{i\hbar\omega}{2}$. Then, equation (3) becomes

$$\widehat{H}_0 = \left(\omega\sqrt{\frac{m}{2}}\hat{x} + i\frac{\hat{p}}{\sqrt{2m}}\right)\left(\omega\sqrt{\frac{m}{2}}\hat{x} - i\frac{\hat{p}}{\sqrt{2m}}\right) - \frac{\hbar\omega}{2}. \quad (4)$$

By doing several algebraic steps, it is found

$$\widehat{H}_0 = \hbar\omega\left(\left(\sqrt{\frac{m\omega}{2\hbar}}\hat{x} + i\frac{\hat{p}}{\sqrt{2m\hbar\omega}}\right)\left(\sqrt{\frac{m\omega}{2\hbar}}\hat{x} - i\frac{\hat{p}}{\sqrt{2m\hbar\omega}}\right) - \frac{1}{2}\right). \quad (5)$$

Next, by defining

$$\hat{a} \equiv \sqrt{\frac{m\omega}{2\hbar}}\hat{x} + i\frac{\hat{p}}{\sqrt{2m\hbar\omega}} \;; \qquad \hat{a}^\dagger \equiv \sqrt{\frac{m\omega}{2\hbar}}\hat{x} - i\frac{\hat{p}}{\sqrt{2m\hbar\omega}} \;; \quad (6)$$

where $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and creation operators respectively, then equation (5) can also be written in the form

$$\widehat{H}_0 = \hbar\omega\left(\hat{a}\hat{a}^\dagger - \frac{1}{2}\right), \quad (7)$$

or

$$\widehat{H}_0 = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right), \quad (8)$$

of course by using the form of commutation $[\hat{a}, \hat{a}^\dagger] = 1$. We can also express the explicit form of the position operator $\hat{x}$ and the momentum $\hat{p}$ in the operator notation $\hat{a}$ and $\hat{a}^\dagger$ by inversing equation (6), so that

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}(\hat{a} + \hat{a}^\dagger) \text{ ; and} \qquad \hat{p} = -i\sqrt{\frac{m\hbar\omega}{2}}(\hat{a} - \hat{a}^\dagger). \quad (9)$$

In Dirac notation, the state of the harmonic oscillator system can be expressed in the form $|n\rangle$, which of course fulfills the orthogonality property of $\langle n'|n\rangle = \delta_{n'n}$; where $\delta_{n'n}$ is the Kronecker delta function. If the operators $\hat{a}$ and $\hat{a}^\dagger$ are applied to this state, then we get

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle \text{and} \qquad \hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle . \quad (10)$$

Furthermore, if the annihilation and creation operators are worked on respectively in state $|n\rangle$, then we get

$$\hat{a}^\dagger\hat{a}|n\rangle = n|n\rangle. \quad (11)$$

A new operator of the form $\widehat{N} \equiv \hat{a}^\dagger\hat{a}$ can be defined from equation (11). The $\widehat{N}$ operator is called the number operator with the property $\widehat{N}|n\rangle = n|n\rangle$, so that from equation (8), the Hamiltonian can be written $\widehat{H}_0 = \hbar\omega\left(\widehat{N} + \frac{1}{2}\right)$. If operator $\widehat{H}_0$ is done in state $|n\rangle$, then $\widehat{H}_0|n\rangle = E_n|n\rangle$, where $E_n = \hbar\omega\left(n + \frac{1}{2}\right)$, with $n = 0, 1, 2, ...$

From equation (10) and using the orthonormality condition for $\{|n\rangle\}$, the elements of the matrix representation for the operators $\hat{a}$ and $\hat{a}^\dagger$ are

$$\langle n'|\hat{a}|n\rangle = \sqrt{n}\delta_{n',n-1} \;; \quad (12)$$

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$$\langle n'|\hat{a}^\dagger|n\rangle = \sqrt{n+1}\delta_{n',n+1} \ .$$

Therefore, the elements of the matrix representation for $\hat{x}$and$\hat{p}$ are

$$\langle n'|\hat{x}|n\rangle = \sqrt{\frac{\hbar}{2m\omega}}\left(\sqrt{n}\delta_{n',n-1} + \sqrt{n+1}\delta_{n',n+1}\right) \ , \tag{13}$$

$$\langle n'|\hat{p}|n\rangle = -i\sqrt{\frac{m\hbar\omega}{2}}\left(\sqrt{n}\delta_{n',n-1} - \sqrt{n+1}\delta_{n',n+1}\right) \ . \tag{14}$$

Finally, we can apply the creation operator to state $|0\rangle$ consecutively to get the state to a particular $|n\rangle$, namely[1]

$$|n\rangle = \left(\frac{(\hat{a}^\dagger)^n}{\sqrt{n!}}\right)|0\rangle. \tag{15}$$

MATRIX REPRESENTATION

In the algebraic analysis, it is necessary to represent the matrix of the operators in a quantum system. In general, it is intended to obtain an explicit expectation value from the quantum system corresponding to the operator that represents it. The information obtained is also of course very useful for further analysis.

This paper analyzes a harmonic oscillator system subjected to QQQ perturbation. Before analyzing the perturbation terms, the following shows a representation of the Hamiltonian matrix of a pure harmonic oscillator, namely

$$\hat{H}_0 \doteq \hbar\omega\begin{pmatrix} 1/2 & 0 & 0 & 0 & 0 & \dots \\ 0 & 3/2 & 0 & 0 & 0 & \dots \\ 0 & 0 & 5/2 & 0 & 0 & \dots \\ 0 & 0 & 0 & 7/2 & 0 & \dots \\ 0 & 0 & 0 & 0 & 9/2 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \tag{16}$$

with bases[17]

$$|0\rangle \doteq \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix}; |1\rangle \doteq \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ \vdots \end{pmatrix}; \quad |2\rangle \doteq \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix}; \dots. \tag{17}$$

The system under study has a Hamiltonian of the form

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 + \beta_1\hat{x}^2 + \beta_2\hat{x}^4 + \beta_3\hat{x}^5, \tag{18}$$

where the third, fourth, and fifth terms of the right side of equation (18) are the perturbation terms; β_1, β_2, andβ_3 are parameters that have different units, and these constants havethe definition

$$\beta_1 = 2\lambda_1 m\omega^2; \quad \beta_2 = 4\lambda_2\frac{m^2\omega^3}{\hbar}; \quad \beta_3 = \lambda_3\left(\frac{2m\omega^{\frac{7}{5}}}{\hbar^{\frac{3}{5}}}\right)^{5/2}, \tag{19}$$

where λ_1, λ_2, and λ_3 are perturbation parameters. Especially for the first term and the second term on the right side of equation (18), it can be expressed in the form of the ladder operators $\hat{a}$ and $\hat{a}^\dagger$ which have a relationship with the $\hat{x}$ operator as shown in equation (9). Therefore, equation (18) can also be expressed in the form of

$$\hat{H} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2}) + \beta_1\hat{x}^2 + \beta_2\hat{x}^4 + \beta_3\hat{x}^5. \tag{20}$$

It should be noted that the term $\hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2})$ is a pure harmonic oscillator operator, while the three extra terms to the right are the perturbation terms, as previously explained.

Now, we begin to analyze the matrix representation of the operators in equation (20). First, we compute the matrix representation for the operator $\hat{a}^\dagger$, with the matrix elements $a^\dagger_{mn}$, i.e.

$$a^\dagger_{mn} = \langle m|\hat{a}^\dagger|n\rangle = \sqrt{n+1}\delta_{m,n+1}. \tag{21}$$

Then, we get the matrix representation of the $\hat{a}^\dagger$ operator, namely

$$\hat{a}^\dagger \doteq \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \dots \\ 1 & 0 & 0 & 0 & 0 & \dots \\ 0 & \sqrt{2} & 0 & 0 & 0 & \dots \\ 0 & 0 & \sqrt{3} & 0 & 0 & \dots \\ 0 & 0 & 0 & 2 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{22}$$

Next, we compute the matrix representation of the $\hat{a}$ operator, with the matrix elements a_{mn}, i.e.

$$a_{mn} = \langle m|\hat{a}|n\rangle = \sqrt{n}\delta_{m,n-1}. \tag{23}$$

Using the bases of equation (17), it is obtained

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$$\hat{a} \doteq \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & \cdots \\ 0 & 0 & \sqrt{2} & 0 & 0 & \cdots \\ 0 & 0 & 0 & \sqrt{3} & 0 & \cdots \\ 0 & 0 & 0 & 0 & 2 & \cdots \\ 0 & 0 & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \qquad (24)$$

By obtaining the matrix representation $\hat{a}^\dagger$and$\hat{a}$, then by using simple algebraic calculations on the relationship $\widehat{H}_0 = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2}\hat{I})$, which is a matrix equation, then we can get the form matrix in equation (16). Note that the existence of $\hat{I}$ is only an identity matrix that corresponds to the matrix order $\hat{a}$and$\hat{a}^\dagger$ so that the matrix calculation process is also suitable.

Now, we come to the perturbation terms. First, we calculate the quadratic perturbation term, which is the matrix representation for $\hat{x}^2$. Based on equations (9) and (10), then

$$\hat{x}^2|n\rangle = \hat{x}(\hat{x})|n\rangle, \qquad (25)$$

$$\hat{x}^2|n\rangle = \sqrt{\frac{\hbar}{2m\omega}}\left(\sqrt{n}\hat{x}|n-1\rangle + \sqrt{n+1}\,\hat{x}|n+1\rangle\right). \qquad (26)$$

By doing some algebraic steps and by multiplying $\langle m|$ from the left to the result of the operation of equation (26), the element of the operator matrix $\hat{x}^2$ is obtained, i.e.

$$\begin{aligned} x_{mn}^2 &= \langle m|\hat{x}^2|n\rangle \\ &= \frac{\hbar}{2m\omega}\left(\sqrt{n(n-1)}\delta_{m,n-2} \right. \\ &\quad + (2n+1)\delta_{m,n} \\ &\quad \left. + \sqrt{(n+1)(n+2)}\delta_{m,n+2}\right). \end{aligned} \qquad (27)$$

So that the matrix representation is obtained, namely

$$\hat{x}^2 \doteq \frac{\hbar}{2m\omega}\hat{A}_1 \qquad (28)$$

where

$$\hat{A}_1 \doteq \begin{pmatrix} 1 & 0 & \sqrt{2} & 0 & 0 & 0 & \cdots \\ 0 & 3 & 0 & \sqrt{6} & 0 & 0 & \cdots \\ \sqrt{2} & 0 & 5 & 0 & 2\sqrt{3} & 0 & \cdots \\ 0 & \sqrt{6} & 0 & 7 & 0 & 2\sqrt{5} & \cdots \\ 0 & 0 & 2\sqrt{3} & 0 & 9 & 0 & \cdots \\ 0 & 0 & 0 & 2\sqrt{5} & 0 & 11 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \qquad (29)$$

Next, we compute the matrix representation for the quartic perturbation term. To obtain a matrix representation of the $\hat{x}^4$ operator, the first thing to do is to calculate $\hat{x}^3|n\rangle = \hat{x}(\hat{x}^2|n\rangle)$, i.e.

$$\begin{aligned} &\hat{x}^3|n\rangle \\ &= \left(\frac{\hbar}{2m\omega}\right)^{\frac{3}{2}}\left(\sqrt{n(n-1)(n-2)}|n-3\rangle \right. \\ &\quad + 3n\sqrt{n}|n-1\rangle + 3(n+1)\sqrt{n+1}|n+1\rangle \\ &\quad \left. + \sqrt{(n+1)(n+2)(n+3)}|n+3\rangle\right). \end{aligned} \qquad (30)$$

Furthermore, we calculate the $\hat{x}^4$ representation, that is

$$\begin{aligned} &\hat{x}^4|n\rangle = \hat{x}(\hat{x}^3|n\rangle) \\ &= \hat{x}\left(\left(\frac{\hbar}{2m\omega}\right)^{\frac{3}{2}}\left(\sqrt{n(n-1)(n-2)}|n-3\rangle \right.\right. \\ &\quad + 3n\sqrt{n}|n-1\rangle + 3(n+1)\sqrt{n+1}|n+1\rangle \\ &\quad \left.\left. + \sqrt{(n+1)(n+2)(n+3)}|n+3\rangle\right)\right). \end{aligned} \qquad (31)$$

By performing several algebraic steps, it is obtained

$$\begin{aligned} &\hat{x}^4|n\rangle \\ &= \left(\frac{\hbar}{2m\omega}\right)^2\left(\sqrt{n(n-1)(n-2)(n-3)}|n \right. \\ &\quad - 4\rangle + 2(2n-1)\sqrt{n(n-1)}|n-2\rangle \\ &\quad + 3(2n^2+2n+1)|n\rangle \\ &\quad + 2(2n+3)\sqrt{(n+1)(n+2)}|n+2\rangle \\ &\quad \left. + \sqrt{(n+1)(n+2)(n+3)(n+4)}|n+4\rangle\right). \end{aligned} \qquad (32)$$

By multiplying $\langle m|$ from the left side with respect to equation (32) is obtained

$$\begin{aligned} &x_{mn}^4 = \langle m|\hat{x}^4|n\rangle \\ &= \left(\frac{\hbar}{2m\omega}\right)^2\left(\sqrt{n(n-1)(n-2)(n-3)}\delta_{m,n-} \right. \\ &\quad + 2(2n-1)\sqrt{n(n-1)}\delta_{m,n-2} \\ &\quad + 3(2n^2+2n+1)\delta_{m,n} \\ &\quad + 2(2n+3)\sqrt{(n+1)(n+2)}\delta_{m,n+2} \\ &\quad \left. + \sqrt{(n+1)(n+2)(n+3)(n+4)}\delta_{m,n+4}\right). \end{aligned} \qquad (33)$$

Based on equation (33), the$\hat{x}^4$ operator matrix representation is obtained as follows

$$\hat{x}^4 \doteq \left(\frac{\hbar}{2m\omega}\right)^2\hat{A}_2, \qquad (34)$$

where

$$\hat{A}_2 \doteq \begin{pmatrix} 3 & 0 & 6\sqrt{2} & 0 & 2\sqrt{6} & 0 & \cdots \\ 0 & 15 & 0 & 10\sqrt{6} & 0 & 2\sqrt{30} & \cdots \\ 6\sqrt{2} & 0 & 39 & 0 & 28\sqrt{3} & 0 & \cdots \\ 0 & 10\sqrt{6} & 0 & 75 & 0 & 36\sqrt{5} & \cdots \\ 2\sqrt{6} & 0 & 28\sqrt{3} & 0 & 123 & 0 & \cdots \\ 0 & 2\sqrt{30} & 0 & 36\sqrt{5} & 0 & 183 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \qquad (35)$$

In the following, we compute the matrix representation for the quintic $\hat{x}^5$perturbation, i.e. start by calculating $\hat{x}^5|n\rangle = \hat{x}(\hat{x}^4|n\rangle)$. By applying the $\hat{x}$ operator in equation (9) to equation (32), and by multiplying $\langle m|$ from the left side to the results of this operation, it is obtained

$$\begin{aligned} x_{mn}^5 &= \langle m|\hat{x}^5|n\rangle \\ &= \left(\frac{\hbar}{2m\omega}\right)^{\frac{5}{2}} \Big(\sqrt{n(n-1)(n-2)(n-3)(n-4)}\,\delta_{m,n-5} \\ &+ 5(n-1)\sqrt{n(n-1)(n-2)}\,\delta_{m,n-3} \\ &+ 5(2n^2+1)\sqrt{n}\,\delta_{m,n-1} \\ &+ 5(2n^2+4n+3)\sqrt{n+1}\,\delta_{m,n+1} \\ &+ 5(n+2)\sqrt{(n+1)(n+2)(n+3)}\,\delta_{m,n+3} \\ &+ \sqrt{(n+1)(n+2)(n+3)(n+4)(n+5)}\,\delta_{m,n+5}\Big). \end{aligned} \quad (36)$$

Based on this equation (36), we get the matrix representation for the operator $\hat{x}^5$ of the form

$$\hat{x}^5 \doteq \left(\frac{\hbar}{2m\omega}\right)^{5/2} \hat{A}_3, \quad (37)$$

where

$$\hat{A}_3 \doteq \begin{pmatrix} 0 & 15 & 0 & 10\sqrt{6} & 0 & 2\sqrt{30} & \cdots \\ 15 & 0 & 45\sqrt{2} & 0 & 30\sqrt{6} & 0 & \cdots \\ 0 & 45\sqrt{2} & 0 & 95\sqrt{3} & 0 & 40\sqrt{15} & \cdots \\ 10\sqrt{6} & 0 & 95\sqrt{3} & 0 & 330 & 0 & \cdots \\ 0 & 30\sqrt{6} & 0 & 330 & 0 & 255\sqrt{5} & \cdots \\ 2\sqrt{30} & 0 & 40\sqrt{15} & 0 & 255\sqrt{5} & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \quad (38)$$

Now, it is time to calculate the total Hamiltonian matrix representation of the system. By using the forms in equation (19), equation (20) in the form of a matrix equation can be written

$$\hat{H} \doteq \hbar\omega\hat{\eta} \quad (39)$$

where

$$\hat{\eta} \doteq \hat{a}^\dagger\hat{a} + \frac{1}{2}\hat{I} + \lambda_1\hat{A}_1 + \lambda_2\hat{A}_2 + \lambda_3\hat{A}_3. \quad (40)$$

By using equations (22), (24), (29), (35), (38) and of course the corresponding identity matrices, equation (40) in the matrix representation can be written explicitly

$$\hat{\eta} \doteq \begin{pmatrix} \frac{1}{2}+\lambda_1+3\lambda_2 & 15\lambda_3 & \sqrt{2}\lambda_1+6\sqrt{2}\lambda_2 & 10\sqrt{6}\lambda_3 & 2\sqrt{6}\lambda_2 & 2\sqrt{30}\lambda_3 & \cdots \\ 15\lambda_3 & \frac{3}{2}+3\lambda_1+15\lambda_2 & 45\sqrt{2}\lambda_3 & \sqrt{6}\lambda_1+10\sqrt{6}\lambda_2 & 30\sqrt{6}\lambda_3 & 2\sqrt{30}\lambda_2 & \cdots \\ \sqrt{2}\lambda_1+6\sqrt{2}\lambda_2 & 45\sqrt{2}\lambda_3 & \frac{5}{2}+5\lambda_1+39\lambda_2 & 95\sqrt{3}\lambda_3 & 2\sqrt{3}\lambda_1+28\sqrt{3}\lambda_2 & 40\sqrt{15}\lambda_3 & \cdots \\ 10\sqrt{6}\lambda_3 & \sqrt{6}\lambda_1+10\sqrt{6}\lambda_2 & 95\sqrt{3}\lambda_3 & \frac{7}{2}+7\lambda_1+75\lambda_2 & 330\lambda_3 & 2\sqrt{5}\lambda_1+36\sqrt{5}\lambda_2 & \cdots \\ 2\sqrt{6}\lambda_2 & 30\sqrt{6}\lambda_3 & 2\sqrt{3}\lambda_1+28\sqrt{3}\lambda_2 & 330\lambda_3 & \frac{9}{2}+9\lambda_1+123\lambda_2 & 255\sqrt{5}\lambda_3 & \cdots \\ 2\sqrt{30}\lambda_3 & 2\sqrt{30}\lambda_2 & 40\sqrt{15}\lambda_3 & 2\sqrt{5}\lambda_1+36\sqrt{5}\lambda_2 & 255\sqrt{5}\lambda_3 & \frac{11}{2}+11\lambda_1+183\lambda_2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

Based on equations (39) and (41), if we choose all perturbation parameters $\lambda_1 = \lambda_2 = \lambda_3 = 0$, then the Hamiltonian of the system will return to the Hamiltonian form of a harmonic oscillator without perturbation, as shown in equation (16). This of course also ensures and verifies the correctness of the results of these calculations. Thus, this matrix can then be used as a reference for perturbation analyzes in harmonic oscillator systems up to the fifth-order.

CONCLUSION

An analysis has been carried out to calculate the representation of the Hamiltonian matrix in the Harmonic Oscillator system which is subjected to simultaneous spatial perturbation of the second, fourth, and fifth orders. This Hamiltonian matrix actually contains an infinite number of elements, but in this paper, only 36 explicit elements are shown, which are shown in equation (41). By using the principle of correspondence, it has also been explained that by choosing all perturbation constants to be zero, the Hamiltonian of the equation (41) matrix is reduced to the form of a pure Harmonic Oscillator Hamiltonian matrix (of course by referring to the relationship with equation (39)).

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