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RESEARCH ARTICLE

Solution of Direct and Inverse Dynamic Problem for the Previously Disturbed Dynamical Systems

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Abstract:
The paper deals with developing backgrounds to study and design the controllers for wide class of dynamical system which can be found in various branches of human life. We offer to use the known approach based on the solution of direct and inverse dynamic problems. This approach us to define system motions by external signals which are given to it as well as define these signals by known motion trajectories. Since the used approach operates with the transfer functions apparatus we offer to generalize these functions by taking into account the system's non-zero initial state while performing the Laplace-Carson's transformation. Such an approach gives us the possibility to consider the generalized transfer function as some matrix differential operator which defines free and perturbed system's motions. We study this operator in our pa-per and show the patterns of its determination and implementation. Our study allows us to supplement the definition of direct and inverse dynamic problems and consider the last one as the problem with the several solutions which define the control signal, external efforts and initial conditions. We use them to define the generalized direct and inverse transfer function.

Keywords: Dynamical system, Direct and inverse dynamic problem, Transfer function, Initial states, Matrix methods, Linear differential operator, Laplace-Carson transformation

1. Introduction

It is difficult to imagine the current stage of human development without using control theory developments. These developments are widespread in various branches of industry [1], transportation [2], [3], technology [4], science [5], communications [6], and much more.

Modern developments in control theory are based on some control methods which allow us to study control system motions [7], [8], [9], stability of these motions [10], [11], and design controllers to form the desired motions [12], [13], [14]. The problem of controllers' design is compounded by the presence of external and internal disturbances and/or uncertainties. That is why a lot of methods and approaches to designing closed-loop control systems are developed [15], [16], [17], [18], [19], [20], [21]. One of them is based on the intellectual control paradigm and allows us to design controllers by using neuro, fuzzy, and so on approaches [22], [23], [24]. The main problem with these approaches using is quite a big error which is caused by subjective factors and the main feature of these approaches is not necessary to know control plant parameters and structure.

The next group of methods is based on using a plant model while a closed-loop system is being designed [25], [26]. Various methods of plant modeling can be used: state space equations [27], transfer functions [28]- [29], and so on. This group of control de-sign methods allows us to reduce control error and increase system performance. Moreover, the use of transfer functions gives us the necessary visualization of the designed control system by representing it with a block diagram. Also, many control methods are known to operate with these diagrams and optimize and transform them. That is why the use of transfer functions is convenient enough while the control system is designed.

The main problem with transfer functions' usage is that these functions are defined as some integrodifferential operator with zero initial conditions. It is clear that this fact reduces the area of using these functions and does not allow to design of a control system for plants with a non-zero initial state. We offer to avoid this drawback by generalizing the transfer function definition and then use it design controller which takes into account the plant's initial state.

Our paper organized as follows: at first, we define the generalized transfer function which contrary to conventional way takes into account system initial conditions, then this transfer function is used to model the generalized linear plant which motion starts from non-zero state. At third, we use the defined in such a way the generalized model to design controller transfer function as solution of inverse dynamic problem for known desired plant motion. Contrary to known control approaches the proposed method allows to define control signal which grant the desired motion from the given initial condition as well as define initial conditions to move from which make it possible to reach the desired paths with given control signal. We prove the correctness and benefits of our method in section Results and Discussion where DC motor drive is modeled to transform it motion equations to the desired form. We use the transformed equations to solve the direct dynamic problem and define the generalized transfer function to study plant motions under the known external signals and internal initial conditions. Also, we use these equations to solve inverse dynamic problem and define speed controller transfer function which allows to get the desired drive motions.

2. Method

2.1. The Solution of Direct Dynamic Problem by using the Generalized Transfer Function for the Linear Plant with Exactly-known Parameters

At first, let us consider the generalized single-input single output controllable plant which dynamic is described with ordinary differential equations in normal form

$$\frac{d}{dt}y_j = \sum_{i=1}^{n} a_{ij}y_i; \;\; \frac{d}{dt}y_n = \sum_{i=1}^{n} a_{in}y_i + b_n u_n, \;\; i = 1, ..., n-1, \tag{2.1}$$

here y_i are plant state variables, a_{ij} and b_n are plant factors which are defined as some functions of its parameters and u_n is a control effort, n is the plant dimension.

We think that plant states can be observed by using system state observer. We consider this observer as some dynamical system which motions depends on both observed object dynamic and inner observer dynamic

$$\frac{d}{dt}\hat{y}_j = \sum_{i=1}^{m} g_{ij}\hat{y}_i; \;\; \frac{d}{dt}y_n + \sum_{i=1}^{n} c_{ij}y_i + \sum_{i=1}^{m} d_i u_n, \tag{2.2}$$

where $\hat{y}_i$ are observer state variables and g_{ij}, c_{ij}, and d_i are observer factors, m is the observer dimension.

Here we think that in the most general case the motions of plant and observer can have different dimensions.

We call (2.2) as the differential observability equations which are the generalization of well-known algebraic observability equations and take into account the various filters and other signal transforming devices in the control system of the considered plant.

The usage of differential equations to define plant and observe dynamics rases the problem of taking into account initial conditions for plant and observer. We call plant and observer motions which starts with non-zero initial conditions, as initially-disturbed motions. Plant and observer are called by us as initially-disturbed ones in this case. It is clear that these initial conditions should be considered while the plant dynamic is being studied as well as its motion is being planned.

That is why we complete (2.1) and (2.2) with their initial conditions y_{i0}, $\hat{y}_{i0}$ and rewrite the studied dynamical system as follows

$$\begin{aligned} &\frac{d}{dt}y_j = a_{ij}y_i; \;\; \frac{d}{dt}y_n = \sum_{i=1}^{n} a_{in}y_i + b_n u_n, \;\; y_i(0) = y_{i0}, \;\; i = 1, ..., n-1; \\ &\frac{d}{dt}\hat{y}_j = \sum_{i=1}^{m} g_{ij}\hat{y}_i + \sum_{i=1}^{n} c_{ij}y_i + \sum_{i=1}^{m} d_i u_n, \;\; \hat{y}_i(0) = \hat{y}_{i0}, \;\; j = 1, ..., m. \end{aligned} \tag{2.3}$$

We call (2.3) as the full controlled initially-disturbed plant equations in the normal form. These equations are written down by using known physical laws and dependencies and they allows us to study the real physical processes in the considered plant by its observable variables.

Let us apply Laplace-Carson transformation to plant and observer state variables

$$\begin{aligned} y_i(t) \to Y_i(s); \;\; \hat{y}_i(t); \to \hat{Y}_i(s); \;\; u_n(t) \to U_n(s); \\ \frac{d}{dt}y_i(t) \to sY_i(s) - sy_{i0}; \;\; \frac{d}{dt}\hat{y}_i(t); \to \hat{Y}_i(s) - s\hat{y}_{i0}, \end{aligned} \tag{2.4}$$

here s is a Laplace operator.

One can use transformations (2.4) to rewrite (2.3) in the operator form

$$\begin{aligned} sY_i(s) - sy_{i0} &= \sum_{i=1}^{n} a_{ij}Y_i(s), \;\; i = 1, ..., n-1; \\ sY_n(s) - sy_{n0} &= \sum_{i=1}^{n} a_{in}Y_i(s) + b_n U_n(s); \\ s\hat{Y}_j - s\hat{y}_{j0} &= \sum_{i=1}^{m} g_{ij}\hat{Y}_i(s) + \sum_{i=1}^{n} c_{ij}Y_i(s) + \sum_{i=1}^{m} d_i U_n(S), \;\; j = 1, ..., m. \end{aligned} \tag{2.5}$$

It is clear that contrary to known control methods and approaches, which are based on usage of plant motions equations in the operator form, equations (2.5) allows us to take into account initial states of plant and observer. The use of these initial states allows us to study, plan, and control of plant motions in the correct way. At the same time, the use of classical approaches to design control signals for initially-disturbed plant can cause reducing stability level for the plant closed-loop control system.

One can use (2.5) in several ways.

The most trivial one to model the plant motion. In this case we assume that plant parameters and input signal are known and plant state variables can be defined as solution of the first two equation of system (2.5).

Let us write down these equations in matrix form as follows

$$sY = aY + bU + sy_0, \tag{2.6}$$

where Y is a n-th sized vector of plant state variables, y_0 is a n-th sized vector of plant initial states, a is nxn-th sized matrix of plant factors which defined its free motion, and b is n-th sized vector of plant controlled-motion factors. We use capitalized letters here to address Laplace-Carson transformation for plant variables and the Laplace operator is skipped to improve formulas reading.

One can use (2.6) to define vector of plant state variables as a result of operator equation solution

$$Y = (sE - a)^{-1} bU + (sE - a)^{-1} sy_0 = (sE - a)^{-1}(bU + sy_0), \tag{2.7}$$

here power -1 means inverse matrix and E is an identity matrix.

Analysis of (2.7) allows us to make an obvious conclusion: plant motions depend on input signal as well as initial conditions. Furthermore, the similarity of the first and second summands in (2.7) allows us to consider initial conditions as an additional input signal and define generalized plant input signal as follows

$$V_2 = \begin{pmatrix} bU & sy_0 \end{pmatrix}^T, \tag{2.8}$$

The generalized control signal allows us to rewrite matrix plant motion equation in such a way

$$\begin{aligned} Y &= (sE - a)^{-1} E_2 V_2, \\ E_2 &= \begin{pmatrix} E & E \end{pmatrix}. \end{aligned} \tag{2.9}$$

One can use (2.9) to define following matrix transfer function

$$W(s) = \frac{Y(s)}{V_2(s)}(sE - a)^{-1} E_2. \tag{2.10}$$

We call this transfer function as the full generalized plant transfer function and consider it as nx2n-th dimensional differential operator. One can use this operator to define plant motions which depends on control inputs and initial states. It is necessary to say that the extension of control inputs vector V_2 (2.8) can cause wrong thinks about possibility to control plant by changing components of vector sy_0 during the plant operating time. To prevent this possible misunderstanding, we claim that the components of initial states vector sy_0 are defined only as scaled Heaviside step functions.

The next problem one can solve by using (2.5) is determination of the observer outputs. To solve this problem let us rewrite the observer equation which is the third equation in (2.5) in matrix form

$$s\hat{Y} = g\hat{Y} + cY + dU + s\hat{y}_0 \tag{2.11}$$

where $\hat{Y}$ and $\hat{y}_0$ are m-th sized vectors of observer state variables and its initial states, g is a mxm-th sized matrix of observer free motion factors, c is a mxn-th sized matrix of plant-observer data transmission channels, d is a m-th sized matrix of observer factors.

The solution of (2.11) is similar to (2.7)

$$\hat{Y} = (sE - g)^{-1}(cY + dU + s\hat{y}_0). \tag{2.12}$$

It is clear that the observer state vector $\hat{Y}$ depends on the plant state vector Y. Let us exclude this vector from our formulas by substituting (2.7) into (2.12)

$$\hat{Y} = (sE_{mm} - g)^{-1}([c(sE_{nn} - a)^{-1} b + d]U + c(sE_{nn} - a)^{-1} sy_0 + s\hat{y}_0). \tag{2.13}$$

here E_{nn} and E_{mm} means nxn and mxm sized identity matrices.

Analysis of (2.13) allows us to claim that the observer motion is defined by input plant signal and initial conditions of plant and observer. This quite trivial statement allows us to set up observer in a correct way and avoid its free motions during synchronizing plant and observer at the beginning of control system motion. It is clear that this fact can improve control system stability.

If one consider dynamical system plant-observer as subordinate multi-loop dynamical system, he can generalizing of (2.13) and make following conclusions to define output of i-th dynamical loop:

1. output vector depends on control signals and as many as i vectors of initial conditions;
2. the motion of i-th loop depends on this loop characteristic polynomial;
3. differential operator near U is defined by control dynamic components for each i-th loop;
4. differential operator near initial condition vector equal to s for i-th loop and take into account loops interrelations for inner loops.

One can use these conclusions as the patterns to define the output for i-th dynamical loop. It is necessary to say that we consider (2.13) as the extended form of expression for i-th loop output but it can be defined in the compact form similar to (2.9) as well

$$\begin{aligned} \hat{Y} &= (sE_{mm} - g)^{-1} E_3 V_3, \\ E_3 &= \begin{pmatrix} E_{mm} & E_{mm} & E_{mm} \end{pmatrix}. \end{aligned} \tag{2.14}$$

The generalized input vector V_3 can be written down in extended form

$$V_3 = \begin{pmatrix} [c(sE_{nn} - a)^{-1} b + d] U & c(sE_{nn} - a)^{-1} s y_0 & s\hat{y}_0 \end{pmatrix}. \tag{2.15}$$

and recursive one

$$V_3 = \begin{pmatrix} c(sE_{nn} - a)^{-1} E_2 V_2 & dU & s\hat{y}_0 \end{pmatrix}. \tag{2.16}$$

Analysis of (2.14)-(2.16) and comparison these expression with (2.8)-(2.9) allows us to define following extended

$$Y_i = \prod_{j=1}^{i} (c_j (sE_{jj} - g_j)^{-1} + d_j) U + \sum_{j=1}^{i} (\prod_{k=2}^{i} (c_j (sE_{kk} - g_k)^{-1}) s y_{j0}, \tag{2.17}$$

where c_i is a factor vector which defines interrelations between loops and d_i is a vector of input factors for i-th loop, E_{ii} is an identity matrix of ixi size,

and recursive expressions to define components of i-th loop output vectors

$$\begin{aligned} Y_i &= (sE_{ii} - g_i)^{-1} E_i V_i, \quad E_i = \begin{pmatrix} E_{ii} & E_{ii} & \dots & E_{ii} \end{pmatrix}; \\ V_i &= (c_i (sE_{(i-1)(i-1)} - g_{i-1})^{-1} E_{i-1} V_{i-1} d_i U s y_{i0}). \end{aligned} \tag{2.18}$$

The solution of (2.17) and (2.18) allows us to define motions of i-th loop in the considered dynamical system. Moreover, one can use these formulas to define the generalized mx(im)-th dimensional dynamical operator which can be considered as generalized transfer function for i-th loop

$$W_i(s) = \frac{Y_i}{V_i} (sE_{ii} - g_i)^{-1} E_i. \tag{2.19}$$

The transfer functions (2.19) give us possibility to define i components of the i-th loop motions which are caused by input signal and non-zero initial states. One can use following matrix transfer functions to define each of these components

$$\begin{aligned} W_{iu}(s) &= \frac{Y_{iu}}{U} = \prod_{j=1}^{i} (c_j (sE_{jj} - g_j)^{-1} + d_j); \\ W_{ij}(s) &= \frac{Y_{ij}}{y_{j0}} = \prod_{k=2}^{j} (sE_{kk} - g_k)^{-1} s. \end{aligned} \tag{2.20}$$

In this case resulted motions can be defined as the sum of the above-mentioned motions, which are defined with (2.20). This fact allows us to claim, that any linear dynamical system with non-zero initial conditions can be described with the following matrix operator expression

$$Y_i = W_{iu}(s) U + \sum_{j=1}^{i} W_{ij}(s) y_{0j}. \tag{2.21}$$

Let us simplify (2.21). We take into account that components of all initial state vectors are weighted Heaviside step functions and define some equivalent transfer function for initial state channel as follows

$$W_{ie}(s) = y_{0q}^{-1} \sum_{j=1}^{i} W_{ij}(s) y_{0j}. \quad (2.22)$$

here y_{0q} is a q-th non-zero vector of initial states.

The use of (2.22) gives us possibility to rewrite (2.21) as follows

$$Y_i = W_{iu}(s)U + W_{ie}(s)y_{0q}. \quad (2.23)$$

Thus, the use of equivalent transfer function (2.22) makes it possible to define motion in the i-th loop as the linear combination of the control signal U and only one vector of initial states y_{0q} which are weighted by some differential matrix operator $W_{iU}(s)$ and $W_{ie}(s)$.

2.2. The Solution of Inverse Dynamic Problem by using the Generalized Transfer Function for the Linear Plant with Exactly-known Parameters

Since we define dynamical system output by known input signals one can consider the both of the above-shown problem as the solutions of direct dynamic problem. Now let us consider the use of above-given expressions to solve inverse dynamic solution which we call as the third way of using (2.5).

At first, we show the main peculiarity of the inverse problem solutions for dynamical systems with non-zero initial states by using expression (2.21) and (2.23). Then we consider these solution more detail by using equations like (2.6).

The classical solution of inverse dynamic problem assumes that the plant motion Y is known and it is necessary to define control signal which caused this motion. As it is mentioned before the dynamic of considered system depends on both control signal and initial state. That is why we offer to specify we inverse dynamic problem and consider its solution for both control signal U and initial state.

We call the solution for control signal as the first kind of inverse dynamic problem solution. Here we think that plant motion and its initial state are known and we define control signal which guarantees plant motion by known paths from known initial state. In the general case this solution can be found if one solves (2.21)(for U

$$U = W_{iU}^{-1}(s)(Y_i - \sum_{j=1}^{i} W_{ij}(s) y_{0j}). \quad (2.24)$$

If one finds similar solution for (2.23), he can rewrite (2.24) in such a way

$$U = W_{iU}^{-1}(s)(Y_i - W_{ie}(s) y_{0q}). \quad (2.25)$$

Thus, the use of equivalent transfer function give us possibility to rewrite (2.24) in compact form (2.25)

Contrary to classical approach the taking into account plant initial states allows us to define the initial state from which plant moves by governing of some control signal. We call the solution of this problem as the second kind of inverse dynamic problem solution and we use following expression as the mathematical solution of this problem

$$y_{0q} = W_{iq}^{-1}(s)(Y_i - W_{iU}(s)U - \sum_{j=1}^{Q-i} W_{ij}(s) y_{0j} - \sum_{j=q+1}^{i} W_{ij}(s) y_{0j}), \quad (2.26)$$

here y_{0q} is a initial state vector for q-th loop of the considered dynamical system.

Analysis of (2.26) shows that only one initial state vector can be defined at once. Other initial state vectors should be defined or know. That is why we offer to solve this kind of inverse dynamic problem as follows

$$W_{ie}(s)y_{0q} = \sum_{j=1}^{i} W_{ij}(s)y_{0j} = Y_i - W_{iU}(s)U. \tag{2.27}$$

Such a solution makes a strong sense because the components of state vectors can be considered as the scaling factors due to their constancy for all time range except the first one. Thus, the expression (2.27) can be rewritten as follows

$$\begin{gathered} W_{ie}^{'}(s)1(s)sigmy_{0q} = \sum_{j=1}^{i} W_{ij}(s)y_{0j} = Y_i - W_{iU}(s)U, \\ W_{ie}^{'}(s) = W_{ie}(s)|y_{0q}|, \end{gathered} \tag{2.28}$$

where $1(s)$ is a i-th size vector of Heaviside functions.

Now we consider solutions of the first and second kinds inverse problems for the generalized linear dynamical system which is given by as follows

$$sY = aY + bU + fsy_0, \tag{2.29}$$

here f is a nxn-th sized weight matrix of components of initial states vector.

The solution of (2.29) for U and y_0 are trivial

$$\begin{aligned} U &= b^{-1}((sE - a)Y - fsy_0); \\ y_0 &= f^{-1}((sE - a)Y - bU)s^{-1}. \end{aligned} \tag{2.30}$$

The similar solution can be written down if one takes into account the generalizing vector input signal

$$V = \begin{pmatrix} U & y_0 \end{pmatrix}^T. \tag{2.31}$$

and rewrite (2.29) in such a way

$$sY = aY + KE_2V, \tag{2.32}$$

where

$$K = \begin{pmatrix} b & sf \end{pmatrix}. \tag{2.33}$$

The use of (2.32) and (2.33) makes it possible to redefine the generalizing vector input signal as follows

$$V = (KE_2)^{-1}(sE - a)Y. \tag{2.34}$$

It is clear that due to the general form of (2.29) the expressions (2.30) and (2.34) defines the most general solution of the first and second kinds inverse dynamic solution. It is necessary to say that the use of equations like (2.32) gives us possibility to solve both of inverse problems at the same time. The main feature of this solution is using of non-square inverse matrix.

Let us specify these solutions for the above-considered plant (2.6) and observer (2.13) dynamical systems.

We start our studies from the plant dynamic study. It is clear that the control signal and initial state vector can be defined by using (2.30) with assumption that f is a nxn-th sized identity matrix

$$\begin{aligned} U &= b^{-1}((sE - a)Y - sy_0); \\ y_0 &= s^{-1}((sE - a)Y - bU). \end{aligned} \tag{2.35}$$

If one takes into account formulas (2.31)-(2.34), he can rewrite (2.35) as follows

$$\begin{aligned} U &= b(b^2 + f^2 s^2)^{-1}(sE - a)Y; \\ y_0 &= fs(b^2 + f^2 s^2)^{-1}(sE - a)Y. \end{aligned} \tag{2.36}$$

Now let us turn our attention to study the observer dynamic. Since we consider the most generalized observer, we can claim that its dynamic depends on the observer structure and its initial state $\hat{y}_0$ as well as external observed signals Y and external control plant signal U. This fact allows us to generalize inverse dynamic problem solution for the class of subordinate dynamical system and claim that one can use motion equation of some dynamical system to define one external signal by motion trajectory of the above-mentioned system and other external signals. Let us solve (2.11) for external signals

$$\begin{aligned} Y &= c^{-1}((sE - g)\hat{Y} - dU - s\hat{y}_0); \\ U &= d^{-1}((sE - g)\hat{Y} - cY - s\hat{y}_0); \end{aligned} \tag{2.37}$$

and vector of initial states

$$\hat{y}_0 = s^{-1}((sE - g)\hat{Y} - cY - dU); \tag{2.38}$$

It is quite clear that signals (2.37) and (2.38) are defined by using vector of observer state variables and others input signals. In case, when the use of others signals is not desired, one can rewrite (2.37) and (2.38) in form similar to (2.36)

$$\begin{aligned} Y &= c(c^2 + d^2 + s^2)^{-1}(sE - g)\hat{Y}; \\ U &= d(c^2 + d^2 + s^2)^{-1}(sE - g)\hat{Y}; \\ y_0 &= s(c^2 + d^2 + s^2)^{-1}(sE - g)\hat{Y}. \end{aligned} \tag{2.39}$$

Analysis of the above-given expressions shows that one can solve the inverse dynamic problem by using both matrix and transfer function approaches. Since these approaches describe motions of the same dynamical system, one get similar results by using various approaches.

3. Results and Discussion

3.1. DC Series Electric Drive Modeling

Let us consider the linearized differential equation of DC series electric drive.

$$\begin{aligned} \frac{d}{dt}\omega &= -\frac{h}{J}\omega + \frac{KK_\phi I_{nom}}{J}I_a - \frac{1}{J}T_c; \\ \frac{d}{dt}I_a &= \frac{KK_\phi I_{nom}}{T_a R_a}\omega - (\frac{1}{T_a} + \frac{KK_\phi \omega_{nom}}{T_a R_a})I_a + \frac{1}{R_a T_a}U_a, \end{aligned} \tag{3.40}$$

where ω is a DC motor speed, I_a is a DC motor current, h is a friction factor, K is a constructive factor, K_ϕ is a linearization factor, J is a DC drive inertia, T_c is a DC drive load torque, I_{nom} and ω_{nom} is a nominal current and speed of DC drive, R_a is an armature resistance, U_a is a DC voltage, T_a is an electromagnetic constant

$$T_a = \frac{L_a}{R_a}, \tag{3.41}$$

where L_a is an armature inductance.

We simplify (3.40) by taking into account (3.41) and following factors

$$\begin{aligned} a_{11} &= -\frac{h}{j}; & a_{12} &= \frac{KK_\phi I_{nom}}{J}; & m_1 &= -\frac{1}{j}; \\ a_{21} &= \frac{KK_\phi I_{nom}}{T_a R_a}; & a_{22} &= -\frac{1}{T_a} - \frac{KK_\phi \omega_{nom}}{T_a R_a}; & m_2 &= \frac{1}{R_a T_a} \end{aligned} \tag{3.42}$$

as follows

$$\frac{d}{dt}\omega = a_{11}\omega + a_{12}I_a + m_1T_c;$$
$$\frac{d}{dt}I_a = a_{21}\omega + a_{22}I_a + m_2U_a, \tag{3.43}$$

Let as assume that the considered electric drive operates with some speed $\omega(0)$ and current $I_a(0)$ before its dynamic studying started. Moreover, we think that it is impossible to use any speed sensors in our electric drive due to mechanical restrictions and that is why we observe the DC speed by using current sensor with gain K_I and voltage sensor with gain K_U. The outputs of both of sensors are filtered by using the simplest RC filter. These filters outputs can be defined in the following way

$$\frac{d}{dt}\hat{u}_i = -\frac{1}{R_IC_I}\hat{u}_i + \frac{K_I}{R_IC_I}I_a;$$
$$\frac{d}{dt}\hat{u}_U = -\frac{1}{R_UC_U}\hat{u}_U + \frac{K_U}{R_UC_U}U_a, \tag{3.44}$$

here R_U, R_I, C_U, and C_I are filters parameters.

Thus, we avoid noising of current and voltage signals by replacing real state variables I_a and U_a with filtered ones $\hat{u}_i$ and $\hat{u}_U$.

The second Kirchhoff rule for the DC armature circuit in steady state allows us to define DC speed in such a way

$$\omega = \frac{U_a - I_aR_a}{KK_\phi I_{nom}}. \tag{3.45}$$

If one takes into account filtered variables of DC electric drive, he can rewrite (3.45) in such a way

$$\omega = \frac{1}{KK_\phi}\frac{\hat{u}_U}{K_U} - \frac{R_a}{KK_\phi}\frac{\hat{u}_i}{K_I}. \tag{3.46}$$

Let us rewrite (3.46) by taking into account following factors

$$d_1 = \frac{1}{KK_\phi K_U}; \quad c_1 = -\frac{1}{KK_\phi K_I} \tag{3.47}$$

as follows

$$\hat{\omega} = d_1\hat{u}_U + c_1\hat{u}_i. \tag{3.48}$$

We consider (3.44) and (3.48) with factors (3.47) as speed observer equations with observer state vector

$$\hat{\omega} = \begin{pmatrix}\hat{\omega} & \hat{u}_I & \hat{u}_U\end{pmatrix}. \tag{3.49}$$

Since we assume that DC drive operates before its studying is being started, we think that state vector (3.49) has non-zero initial state

$$\hat{\omega}(0) = \begin{pmatrix}\hat{\omega}(0) & \hat{u}_I(0) & \hat{u}_U(0)\end{pmatrix}. \tag{3.50}$$

We consider DC drive equations (3.43) and observer equations (3.44) and (3.48) as the differential-algebraic equations of the dynamical system "control plant-observer" with initial condition vector (3.50).

Since the above-given theoretical backgrounds operates with differential equations let us rewrite observer equation (3.48) into differential form

$$\frac{d}{dt}\hat{\omega} = a_{52}I_a + a_{53}\hat{u}_i + a_{54}\hat{u}_U + m_5U_a, \tag{3.51}$$

where

$$a_{52} = \frac{c_1 K_I}{R_I C_I}; \quad a_{53} = -\frac{c_1}{R_I C_I}; \quad a_{54} = -\frac{d_1}{R_U C_U}; \quad m_5 = -\frac{d_1 K_U}{R_U C_U} \tag{3.52}$$

We take into account Laplace-Carson transformation for linear differential equations with non-zero initial states and rewrite the system which dynamic is defined by (3.44), (3.48) with (3.47) and (3.51) with (3.52) into operator form

$$\begin{aligned} s\hat{\omega} &= a_{11}\omega + a_{12}I_a + m_1 T_c + s\hat{\omega}(0); \\ sI_a &= a_{21}\omega + a_{22}I_a + m_2 U_a + sI_a(0); \\ s\hat{u}_i &= a_3\hat{u}_i + a_{32}I_a + s\hat{u}_i(0); \\ s\hat{u}_U &= a_{44}\hat{u}_U + m_4 U_a + s\hat{u}_U(0); \\ s\hat{\omega} &= a_{52}I_a + a_{53}\hat{u}_i + a_{54}\hat{u}_U + m_5 U_a + s\hat{\omega}(0). \end{aligned} \tag{3.53}$$

Analysis of (3.53) shows that the use of observer cause to considering of studied DC electric drive with speed observer as multichannel dynamical system.

Let us rewrite (3.53) into matrix form

$$sY = AY + MU + sY, \tag{3.54}$$

here

$$\begin{aligned} Y &= \begin{pmatrix} \omega & I_a & \hat{u}_i & \hat{u}_U & \hat{\omega} \end{pmatrix}^T; \ \mathbf{U} = \begin{pmatrix} T_c & U_a \end{pmatrix}^T; \\ \mathbf{Y_0} &= \begin{pmatrix} \omega(0) & I_a(0) & \hat{u}_i(0) & \hat{u}_U(0) & \hat{\omega}(0) \end{pmatrix}^T. \end{aligned} \tag{3.55}$$

$$A = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \\ 0 & 0 \\ 0 & m_4 \\ 0 & m_5 \end{bmatrix}; A = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 \\ 0 & a_{32} & a_{33} & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 \\ 0 & a_{52} & a_{53} & a_{54} & 0 \end{bmatrix}$$

One can use (3.54) with (3.55) to study the considered system dynamic as well as its steady state, define its frequency responses, design controller and more.

3.2. Solution of Direct Dynamc Problem

Let us solve direct dynamic problem for the system (3.54). It is understood that this solution in the matrix form can be written down as follows. Matrix expression (56) allows us to define full state space vector Y which components depend on input signals vector U, initial state vector Y0, and system parameters matrices A and M.

$$Y = (sE - A)^{-1}(MU + sY_0) \tag{3.56}$$

Matrix expression (56) allows us to define full state space vector Y which components depend on input signals vector U, initial state vector Y_0, and system parameters matrices A and M.

$$
\begin{aligned}
\omega &= \frac{m_1 s T_c - m_1 a_{22} T_c + a_{12} m_2 U_a + s^2 \omega(0) - a_2 2 s \omega(0) + a_1 2 s I_a(0)}{q_{11} q_{22} - q_{12} a_{21} - (a_{11} + q_{22}) s + s^2};\\
I_a &= \frac{a_{21} m_1 T_c + m_2 s U_a - a_{11} m_2 U_a + q_{21} s \omega(0) + s^2 I_a(0) - a_{11} s I_a(0)}{q_{11} q_{22} - q_{12} a_{21} - (a_{11} + q_{22}) s + s^2};\\
\widehat{u_i} &= \frac{a_{32} a_{21} m_1 T_c - a_{32} m_2 s U_a + a_{11} a_{32} m_2 s U_a + q_{32} a_{21} s \omega(0) - a_{32} s^2 I_a(0) + a_{11} a_{32} s I_a(0)}{(a_{12} a_{21} - a_{11} a_{22}) a_{33} + (a_{11} a_{22} + a_{11} a_{33} + a_{22} a_{33} - a_{12} a_{21}) s - (a_{11} + a_{22} + a_{33}) s^2 + s^3} + \frac{s \widehat{u_i}(0)}{s - a_{33}};\\
\hat{\omega} &= \frac{a_{21} \left(a_{32} a_{53} - a_{33} a_{52} + a_{52} s\right) m_1 T_c + a_{21} \left(a_{32} a_{53} - a_{33} a_{52} + a_{52} s\right) \omega\left(0\right)}{\left(a_{12} a_{21} - a_{11} a_{22}\right) a_{33} + \left(a_{11} a_{22} + a_{11} a_{33} + a_{22} a_{33} - a_{12} a_{21}\right) s - \left(a_{11} + a_{22} + a_{33}\right) s^2 + s^3} -\\
&- \frac{(a_{11} a_{32} a_{53} - a_{11} a_{33} a_{52} + \left(a_{11} a_{52} - a_{32} a_{53} + a_{33} a_{52}\right) s - a_{52} s^2) I_a\left(0\right)}{\left(a_{12} a_{21} - a_{11} a_{22}\right) a_{33} + \left(a_{11} a_{22} + a_{11} a_{33} + a_{22} a_{33} - a_{12} a_{21}\right) s - \left(a_{11} + a_{22} + a_{33}\right) s^2 + s^3} -\\
&- \frac{(a_{11} a_{32} a_{53} - a_{11} a_{33} a_{52} + \left(a_{11} a_{52} - a_{32} a_{53} + a_{33} a_{52}\right) s - a_{52} s^2) m_2 U_a}{\left(a_{12} a_{21} - a_{11} a_{22}\right) a_{33} s + \left(a_{11} a_{22} + a_{11} a_{33} + a_{22} a_{33} - a_{12} a_{21}\right) s^2 - \left(a_{11} + a_{22} + a_{33}\right) s^3 + s^4} +\\
&+ \frac{a_{53} u_i\left(0\right)}{s - a_{33}} + \frac{a_{54} m_4 U_a}{s^2 - a_{44} s} + \frac{\hat{u}_U\left(0\right)}{s - a_{44}} + \frac{m_5 U_a}{s} + s \hat{\omega}\left(0\right)
\end{aligned}
\tag{3.57}
$$

Analysis of (57) allows us to study effect of each input signal. Contrary to well-known control approach, which is based on transfer function usage, out approach allows to take into account system initial state while its dynamic is being studied. If one consider the first and second expression of (57) in details he finds well-known summands, which defines input signals influence, in these expressions.

Nevertheless, the analysis of full system shows that its multichannel nature causes different transfer functions as well as different characteristic polynomial for various state variables. Moreover, observed speed equation dramatically differs unobserved one. From one hand it causes the necessity to check stability of each channel of the considered system, from another one it makes possibility to effect on system output variable in a wide range by defining observer parameters.

The last three expressions define observer dynamic as the combinations of system input signals and its initial states. Moreover, it is clear that the dynamic of observer subsystem depends on initial states of both DC drive and observer.

This fact makes observer studies more complex. To simplify the study of the considered observer dynamic let us define its motions as the function of input signals, initial conditions and DC drive state variable

$$
\begin{aligned}
s \hat{u}_i &= a_{33 \hat{u}_i + b_{32} I_a + s \hat{u}_i(0)};\\
s \hat{u}_U &= a_{44} \hat{u}_U + m_4 U_a + s \hat{u}_U\left(0\right);\\
s \hat{\omega} &= b_{52} I_a + a_{53} \hat{u}_i + a_{54} \hat{u}_U + m_5 U_a + s \hat{\omega}\left(0\right),
\end{aligned}
\tag{3.58}
$$

here

$$
b_3 = a_{32}; \quad b_5 = a_{52} \tag{3.59}
$$

It is clear that for plant (58) with factors (59) observer dynamic can be defined by the following matrix equation

$$
s \hat{\mathbf{Y}} = \hat{\mathbf{A}} \hat{\mathbf{Y}} + \hat{\mathbf{B}} \hat{\mathbf{Y}} + \hat{\mathbf{M}} U_a + s \hat{\mathbf{Y}}_{\mathbf{0}}, \tag{3.60}
$$

where

$$Y = \begin{pmatrix}\omega & I_a\end{pmatrix}^T; \;\; \hat{\mathbf{Y}} = \begin{pmatrix}\hat{u}_i & \hat{u}_U & \hat{\omega}\end{pmatrix}^T;$$
$$\hat{\mathbf{Y}}_\mathbf{0} = \begin{pmatrix}\hat{u}_i(0) & \hat{u}_U(0) & \hat{\omega}(0)\end{pmatrix}^T;$$
$$\hat{\mathbf{A}} = \begin{pmatrix}a_{33} & 0 & 0\\ 0 & a_{44} & 0\\ a_{53} & a_{54} & 0\end{pmatrix}; \;\; \hat{\mathbf{M}} = \begin{pmatrix}0\\ m_4\\ m_5\end{pmatrix}; \;\; \hat{\mathbf{B}} = \begin{pmatrix}0 & b_{32}\\ 0 & 0\\ 0 & b_{52}\end{pmatrix}. \tag{3.61}$$

Solution of (60) is a clear one

$$\hat{\mathbf{Y}} = \left(\hat{\mathbf{E}}s - \hat{\mathbf{A}}\right)^{-1}\left(\hat{\mathbf{B}}\mathbf{Y} + \hat{\mathbf{M}}U_a + s\hat{\mathbf{Y}}_\mathbf{0}\right). \tag{3.62}$$

If one takes into account (61) he can rewrite (62) in the extended form

$$\hat{u}_i = \frac{b_{32}}{\mathrm{s}-\mathrm{a}_{33}}I_a + \frac{\mathrm{s}}{\mathrm{s}-\mathrm{a}_{33}}\hat{u}_i(0); \;\; \hat{u}_U = \frac{m_4}{\mathrm{s}-\mathrm{a}_{44}}U_a + \frac{\mathrm{s}}{\mathrm{s}-\mathrm{a}_{44}}\hat{u}_U(0);$$
$$\hat{\omega} = \left(\frac{b_{52}}{s} + \frac{a_{53}b_{32}}{s^2 - \mathrm{a}_{33}s}\right)I_a + \left(\frac{m_5}{s} + \frac{a_{54}m_4}{s^2 - \mathrm{a}_{44}s}\right)U_a + \frac{a_{53}}{\mathrm{s}-\mathrm{a}_{33}}\hat{u}_i(0) + \frac{a_{54}}{\mathrm{s}-\mathrm{a}_{44}}\hat{u}_U(0) + \hat{\omega}(0). \tag{3.63}$$

Thus, expressions (57) and (63) allows us to define each system state variable as linear combinations of its variables and initial states which are weighted with some differential operators

$$\omega = \frac{m_1(s-a_{22})}{a_{11}a_{22} - a_{12}a_{21} - (a_{11}+a_{22})s + s^2}T_c + \frac{a_{12}m_2}{a_{11}a_{22} - a_{12}a_{21} - (a_{11}+a_{22})s + s^2}U_a +$$
$$+ \frac{(s-a_{22})s}{a_{11}a_{22} - a_{12}a_{21} - (a_{11}+a_{22})s + s^2}\omega(0) + \frac{a_{12}s}{a_{11}a_{22} - a_{12}a_{21} - (a_{11}+a_{22})s + s^2}I_a(0);$$
$$I_a = \frac{a_{21}m_1}{a_{11}a_{22} - a_{12}a_{21} - (a_{11}+a_{22})s + s^2}T_c + \frac{m_2(s-a_{11})}{a_{11}a_{22} - a_{12}a_{21} - (a_{11}+a_{22})s + s^2}U_a +$$
$$+ \frac{a_{21}s}{a_{11}a_{22} - a_{12}a_{21} - (a_{11}+a_{22})s + s^2}\omega(0) + \frac{(s-a_{11})s}{a_{11}a_{22} - a_{12}a_{21} - (a_{11}+a_{22})s + s^2}I_a(0); \tag{3.64}$$

$$\hat{u}_i = \frac{b_{32}}{\mathrm{s}-\mathrm{a}_{33}}I_a + \frac{\mathrm{s}}{\mathrm{s}-\mathrm{a}_{33}}\hat{u}_i(0); \;\; \hat{u}_U = \frac{m_4}{\mathrm{s}-\mathrm{a}_{44}}U_a + \frac{\mathrm{s}}{\mathrm{s}-\mathrm{a}_{44}}\hat{u}_U(0);$$
$$\hat{\omega} = \frac{b_{52}\mathrm{s} - \mathrm{a}_{33}b_{52} + a_{53}b_{32}}{s^2 - a_{33}s}I_a + \frac{m_5s - m_5a_{44} + a_{54}m_4}{s^2 - \mathrm{a}_{44}s}U_a + \frac{a_{53}}{\mathrm{s}-\mathrm{a}_{33}}\hat{u}_i(0) + \frac{a_{54}}{\mathrm{s}-\mathrm{a}_{44}}\hat{u}_U(0) + \hat{\omega}(0)$$

Let us define differential operators in (64) as components of matrix transfer function

$$W_{11} = \frac{m_1(s-a_{22})}{a_{11}a_{22} - a_{12}a_{21} - (a_{11}+a_{22})s + s^2}; W_{12} = \frac{a_{12}m_2}{a_{11}a_{22} - a_{12}a_{21} - (a_{11}+a_{22})s + s^2} +$$
$$W_{13} = \frac{(s-a_{22})s}{a_{11}a_{22} - a_{12}a_{21} - (a_{11}+a_{22})s + s^2}; W_{14} = \frac{a_{12}s}{a_{11}a_{22} - a_{12}a_{21} - (a_{11}+a_{22})s + s^2};$$
$$W_{21} = \frac{a_{21}m_1}{a_{11}a_{22} - a_{12}a_{21} - (a_{11}+a_{22})s + s^2}; W_{22} = \frac{m_2(s-a_{11})}{a_{11}a_{22} - a_{12}a_{21} - (a_{11}+a_{22})s + s^2};$$
$$W_{23} = \frac{a_{21}s}{a_{11}a_{22} - a_{12}a_{21} - (a_{11}+a_{22})s + s^2}; W_{24} = \frac{(s-a_{11})s}{a_{11}a_{22} - a_{12}a_{21} - (a_{11}+a_{22})s + s^2};$$
$$W_{35} = \frac{b_{32}}{\mathrm{s}-\mathrm{a}_{33}}; W_{36} = \frac{\mathrm{s}}{\mathrm{s}-\mathrm{a}_{33}}; W_{42} = \frac{m_4}{\mathrm{s}-\mathrm{a}_{44}}; W_{47} = \frac{\mathrm{s}}{\mathrm{s}-\mathrm{a}_{44}}; \mathrm{W}_{55} = \frac{b_{52}\mathrm{s} - \mathrm{a}_{33}b_{52} + a_{53}b_{32}}{s^2 - a_{33}s};$$
$$W_{52} = \frac{m_5s - m_5a_{44} + a_{54}m_4}{s^2 - \mathrm{a}_{44}s}; W_{56} = \frac{a_{53}}{\mathrm{s}-\mathrm{a}_{33}}; W_{57} = \frac{a_{54}}{\mathrm{s}-\mathrm{a}_{44}}; W_{58} = 1 \tag{3.65}$$

and take into consideration the generalized input vector

$$\mathbf{V} = \begin{pmatrix} T_c & U_a & \omega(0) & I_a(0) & I_a & \hat{u}_i(0) & \hat{u}_U(0) & \hat{\omega}(0) \end{pmatrix}^T \tag{3.66}$$

as well as the generalized state vector

$$\mathbf{Y} = \begin{pmatrix} \omega & I_a & \hat{u}_i & \hat{u}_U & \hat{\omega} \end{pmatrix}^T \tag{3.67}$$

The use of expressions (65), (66), and (67) allows us to rewrite (64) in terms of the generalized matrix transfer function

$$\mathbf{Y} = \mathbf{W}(s)\,\mathbf{V}, \tag{3.68}$$

here

$$\mathbf{W}(s) = \begin{pmatrix} W_{11} & W_{12} & W_{13} & W_{14} & 0 & 0 & 0 & 0 \\ W_{21} & W_{22} & W_{23} & W_{24} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & W_{35} & W_{36} & 0 & 0 \\ 0 & W_{42} & 0 & 0 & 0 & 0 & W_{47} & 0 \\ 0 & W_{52} & 0 & 0 & 0 & W_{56} & W_{57} & W_{58} \end{pmatrix} \tag{3.69}$$

It is clearly understood that transfer function (69) allows us to simplify the system dynamic describing. This transfer function in a clear way defines interconnections between plant and its observer. That is why drive current Ia is considered in both ways as the state variable in vector Y and input variable for the observer in vector V. This fact increases the size of the considered matrix transfer function. So, if interconnections between the observer and drive is not necessary to study, one can decrease size of the input vector (66) and matrix transfer function (69) if defines them by using (57). In this case input vector can be written down as follows

$$V' = \begin{pmatrix} T_c & U_a & \omega(0) & I_a(0) & \hat{u}_i(0) & \hat{u}_U(0) & \hat{\omega}(0) \end{pmatrix}^T. \tag{3.70}$$

The components of the generalized transfer matrix in the first two rows should not be changed and other components are defined in such a way

$$\begin{aligned}
&W'_{11} = W_{11};\;\; W'_{12} = W_{12};\;\; W'_{13} = W_{13};\;\; W'_{14} = W_{14};\\
&W'_{21} = W_{21};\;\; W'_{22} = W_{22};\;\; W'_{23} = W_{23};\;\; W'_{24} = W_{24};\\
&W'_{31} = \frac{a_{32}a_{21}m_1}{D_3(s)};\;\; W'_{32} = \frac{a_{32}m_2(a_{11}-1)s}{D_3(s)};\;\; W'_{33} = \frac{a_{32}a_{21}s}{D_3(s)};\\
&W'_{34} = \frac{a_{32}s(a_{11}-s)}{D_3(s)};\;\; W'_{35} = \frac{s}{s-a_{33}};\\
&W'_{42} = \frac{m_4}{s-a_{44}};\;\; W'_{46} = \frac{s}{s-a_{44}};\;\; W'_{51} = \frac{a_{21}(a_{32}a_{53}-a_{33}a_{52}+a_{52}s)m_1}{D_3(s)};\\
&W'_{52} = -\frac{(a_{11}a_{32}a_{53}-a_{11}a_{33}a_{52}+(a_{11}a_{52}-a_{32}a_{53}+a_{33}a_{52})s-a_{52}s^2)}{D_3(s)} + \frac{a_{54}m_4}{s^2-a_{44}s} + \frac{m_5}{s};\\
&W'_{53} = \frac{a_{21}(a_{32}a_{53}-a_{33}a_{52}+a_{52}s)}{D_3(s)};\;\; W'_{55} = \frac{a_{53}}{s-a_{33}};\;\; W'_{56} = \frac{1}{s-a_{44}};\;\; W'_{57} = s;\\
&W'_{54} = -\frac{(a_{11}a_{32}a_{53}-a_{11}a_{33}a_{52}+(a_{11}a_{52}-a_{32}a_{53}+a_{33}a_{52})s-a_{52}s^2)}{D_3(s)},
\end{aligned} \tag{3.71}$$

here

$$D_3(s) = (a_{12}a_{21}-a_{11}a_{22})a_{33} + (a_{11}a_{22}+a_{11}a_{33}+a_{22}a_{33}-a_{12}a_{21})s - (a_{11}+a_{22}+a_{33})s^2 + s^3. \tag{3.72}$$

Matrix components (71) allow us to rewrite (69) in such a way

$$W'(s) = \begin{pmatrix} W'_{11} & W'_{12} & W'_{13} & W'_{14} & 0 & 0 & 0 \\ W'_{21} & W'_{22} & W'_{23} & W'_{24} & 0 & 0 & 0 \\ W'_{31} & W'_{32} & W'_{33} & W'_{34} & W'_{35} & 0 & 0 \\ 0 & W'_{42} & 0 & 0 & 0 & W'_{46} & 0 \\ W'_{51} & W'_{52} & W'_{53} & W'_{54} & W'_{55} & W'_{56} & W'_{57} \end{pmatrix} \tag{3.73}$$

and taking into account (70) defines system dynamic as follows

$$Y = W'(s)\,V'. \tag{3.74}$$

In such a way one can consider both of (74) and (68) as the solution of direct dynamic problem in the general case.

3.3. Solution of Inverse Dynamic Problem

It is clear that both of the first and the second kinds inverse dynamic problems can be solved by using (53) and assuming that the components of system state vector are known and input voltage, load torque or initial states should be defined.
It is necessary to say that the number of equations in (53) less than number of input signals and initial states one can solve only one kind of inverse problem at once by using (53). It is quite inconvenient and that is why instead of making solution of (53) and all transformations we remark that we have already found solution for direct dynamic problem as the matrix expressions (74) and (68). The inversion of these expressions allows us to define the generalized input vectors V and V'.

$$V = \mathbf{W}^{-1}(s)\,Y; \quad V' = W'^{-1}(s)\,Y. \tag{3.75}$$

Non-square inverse matrix we define in such a way

$$\begin{aligned} \mathbf{W}^{-1}(s) &= \mathbf{W}^{T}(s)\,(\mathbf{W}(s)\,\mathbf{W}(s))^{-1}; \\ W'^{-1}(s) &= W'^{T}(s)\left(\mathbf{W}'(s)\,\mathbf{W}'^{T}(s)\right)^{-1}. \end{aligned}$$

Due to the complexity of the obtained expressions we do not show them here but claim that the modern mathematical software, which perform symbolic calculations, define them in a simple way.
Analysis of above-given formulas shows that the proposed approach which is based on using the generalized matrix transfer function allows us to solve direct at inverse dynamic problems for any linear dynamical systems with non-zero initial states. Since very formal methods mathematical are used one can easy implement our approach by using modern mathematical software with symbolic calculations like Mathsoft Matlab, Waterloo Maple, SageMath and similar ones.
From our viewpoint the main drawback of our approach is the considering of linear systems with exactly-known constant parameters. Since the parameters of many real technical systems changes during their operation modes, our formulas can be inapplicable here. This drawback can be avoided by using various interval methods which allows us to give some uncertainty to our equations and thus to take into account possible parameters changing.
Also, it should be mentioned about nonlinearity of real technical systems and inapplicability of Taylor-based linearization for these systems. That is why we mark one more possible way to developing our approach is adopting it to nonlinear differential equations and replacing linear differential operators with nonlinear ones as well as using piecewise linear functions to describe system nonlinearities.

4. Conclusions

The generalization of transfer function as the matrix linear differential operator makes it possible to take into account not only system control signal and external disturbances but also allows to consider system initial state. Such system dynamic describing is a very formal and allows in strong mathematical way define system direct and inverse dynamic. Since the proposed approach is based on matrix methods one can use it to operate with wide range single-channel and multi-channel dynamical system and define theirs motions or input signals at once and consider full system motions or motions of each subsystem in which the system can be split. The forces which cause the above-mentioned motions can be defined as well.

References

[1] M. Z. Nd1, C. Wang, D. Zhang, H. Zhuang, and B. Lu, "Coordinated synchronization control of multi-motor system based on synergetic control theory," *2018 Chinese Control And Decision Conference (CCDC)*, pp. 160–164, 2018.

[2] S. Peresada, Y. Nikonenko, S. Kovbasa, A. Kuznietsov, and D. Pushnitsyn, "Rapid prototyping station for batteries supercapacitors hy-brid energy storage systems," *2019 IEEE 39th International Conference on Electronics and Nanotechnology (ELNANO)*, pp. 826–831, 2019.

[3] R. Voliansky, O. Sadovoi, Y. Sokhina, I. Shramko, and N. Volianska, "Sliding mode interval controller for the mobile robot," *2019 XIth International Scientific and Practical Conference on Electronics and Information Technologies (ELIT)*, pp. 76–81, 2019. doi: 10.1109/ELIT.2019.8892330.

[4] S. Peresada, S. Kovbasa, V. Trandafilov, and V. Pyzhov, "Sliding mode observer based control of induction motors: Experimental study," *2014 IEEE International Conference on Intelligent Energy and Power Systems (IEPS)*, pp. 261–265, 2014. doi: 10.1109/IEPS.2014.6874191.

[5] Iswanto and A. Ma'arif, "Rapid prototyping station for batteries supercapacitors hy-brid energy storage systems," *in IEEE Access*, pp. 57003–57011, 2020. doi: 10.1109/ACCESS.2020.2981840.

[6] R. Voliansky, O. Sadovoi, Y. Shramko, N. Volianska, and O. Sinkevych, "Arduino-based implementation of the dual-channel chaotic generator," *2019 3rd International Conference on Advanced Information and Communications Technologies (AICT*, pp. 278–281, 2019. doi: 10.1109/AIACT.2019.8847904.

[7] Z. Wang, R. Lu, and H. Wang, "Finite-time trajectory tracking control of a class of nonlinear discrete-time systems," *in IEEE Transac-tions on Systems, Man, and Cybernetics: Systems*, pp. 1679–1687, July 2017. doi: 10.1109/TSMC.2017.2663523.

[8] H. Chang, X. Su, and S. Wang, "Adaptive trajectory tracking control for a rehabilitative training walker with center of gravity shift," *2020 2nd International Conference on Industrial Artificial Intelligence (IAI)*, pp. 1–2, 2020. doi: 10.1109/IAI50351.2020.9262159.

[9] Q. Ai, Q. Yang, M. Li, X. Feng, and W. Meng, "Implementing multi-dof trajectory tracking control system for robotic arm experimental platform," *2018 10th International Conference on Measuring Technology and Mechatronics Automation (ICMTMA)*, pp. 282–285, 2018. doi: 10.1109/ICMTMA.2018.00075.

[10] D. Yang and Y. Sun, "Siso impedance-based stability analysis for system-level small-signal stability assessment of large scale power electronics-dominated power systems," *in IEEE Transactions on Sustainable Energy*, pp. 537–550, Jan. 2022. doi: 10.1109/TSTE.2021.3119207.

[11] Y.-H. Yang, "Practical stability domain analysis of a class of dynamical system," *2008 International Conference on Machine Learning and Cybernetics*, pp. 2090–2093, 2008. doi: 10.1109/ICMLC.2008.4620750.

[12] S. Higuchi, R. Arimura, C. Premachandra, and K. Kato, "Design of two degree of freedom controller using data conversion method," *2016 16th International Conference on Control, Automation and Systems (ICCAS)*, pp. 1067–1072, 2016. doi: 10.1109/ICCAS.2016.7832442.

[13] R. Voliansky, O. Sadovoi, Y. Sokhina, and N. Volianska, “Active suspension control system,” *2019 IEEE International Conference on Modern Electrical and Energy Systems (MEES)*, pp. 10–13, 2019. doi: 10.1109/MEES.2019.8896419.

[14] B. Verma, S. Sharma, R. Trivedi, and P. K. Padhy, “Controller design for tito process using equivalent transfer function with new rela-tive derivative normalised gain array,” *2018 International Conference on Power Energy, Environment and Intelligent Control (PEE-IC)*, pp. 452–45, 2018. doi: 10.1109/PEEIC.2018.8665609.

[15] Z. Xiu, W. Guo, and W. Wang, “Design of adaptive fuzzy controllers for warship weapon control systems,” *2006 6th World Congress on Intelligent Control and Automation*, pp. 3729–3733, 2006. doi: 10.1109/WCICA.2006.1713067.

[16] M. Yongchao, L. Yungang, and Y. Xuehua, “Quantized output feedback control design for a class of nonlinear systems with unmeasured states dependent growth,” *Proceedings of the 31st Chinese Control Conference*, 2012.

[17] J. Zhang, Y. Kuai, S. Zhou, G. Hou, and M. Ren, “Improved minimum entropy control for two-input and two-output networked control systems,” *2016 UKACC 11th International Conference on Control (CONTROL)*, pp. 1–5, 2016. doi: 10.1109/CONTROL.2016.7737575.

[18] H. H. Tahir and A. A. A. Al-Rawi, “Variable structure control design of process plant based on sliding mode approach,” *2006 Chinese Control Conference*, pp. 932–937, 2006. doi: 10.1109/CHICC.2006.280811.

[19] Y. Liu, S. Zhou, and Z. Hao, “Fuzzy guaranteed cost control design for uncertain chaotic system with polytopic uncertainty,” *Proceedings of the 10th World Congress on Intelligent Control and Automation*, pp. 180–184, 2012. doi: 10.1109/WCICA.2012.6357863.

[20] Y. Man and Y. Liu, “Adaptive stabilizing control design via switching for high-order uncertain nonlinear systems,” *Proceedings of the 32nd Chinese Control Conferencen*, pp. 850–855, 2013.

[21] H. Maghfiroh, M. Nizam, M. Anwar, and A. Ma’Arif, “Improved lqr control using pso optimization and kalman filter estimator,” *in IEEE Access*, pp. 18330–18337, 2022. doi: 10.1109/ACCESS.2022.3149951.

[22] S. Burian, M. Pechinik, M. Pushkar, and A. Tytarenko, “Investigation of the pump unit control system with the neural network produc-tivity estimator,” *2019 IEEE 6th International Conference on Energy Smart Systems (ESS)*, pp. 298–30, 2019. doi: 10.1109/ESS.2019.8764176.

[23] X. Hu, Y. Gong, D. Zhao, and W. Gu, “Structurally optimized neural fuzzy modelling for model predictive control,” *in IEEE Transactions on Industrial Informatics*, pp. 298–30. doi: 10.1109/TII.2021.3133893.

[24] C. Juang and T. B. Bui, “Reinforcement neural fuzzy surrogate-assisted multiobjective evolutionary fuzzy systems with robot learning control application,” *in IEEE Transactions on Fuzzy Systems*, pp. 434–446, March 2020. doi: 10.1109/TFUZZ.2019.2907513.

[25] G. Miloud, S. Hicham, and B. Youcef, “Sensorless speed control of synchronous reluctance motors using model predictive control asso-ciated with model reference adaptive system,” *2021 IEEE 1st International Maghreb Meeting of the Conference on Sciences and Tech-niques of Automatic Control and Computer Engineering MI-STA*, pp. 230–233, 2021. doi: 10.1109/MI-STA52233.2021.9464507.

[26] Z. Leng and Q. Liu, “A simple model predictive control for buck converter operating in ccm,” *2017 IEEE International Symposium on Predictive Control of Electrical Drives and Power Electronics (PRECEDE)*, pp. 19–24, 2017. doi: 10.1109/PRECEDE.2017.8071262.

[27] X.-J. Dong, S. T. Li, N. Jiang, and Y. W. Jing, “Dua-lrate inferential predictive control based on state-space model,” *Proceeding of the 11th World Congress on Intelligent Control and Automation*, pp. 3796–3799, 2014. doi: 10.1109/WCICA.2014.7053349.

[28] N. Paenoi and S. Sitjongsataporn, "Automatic transfer function improvement based on genetic algorithm," *2021 7th International Conference on Engineering, Applied Sciences and Technology (ICEAST)*, pp. 230–233, 2021. doi: 10.1109/ICEAST52143.2021.9426275.
[29] C. P. Basso, "Transfer functions," *in Linear Circuit Transfer Functions: An Introduction to Fast Analytical Techniques , IEEE*, pp. 41–115, 2016. doi: 10.1002/9781119236344.ch02.

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