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RESEARCH ARTICLE

Sombor Indices of Prime Coprime Graph for Integer Modulo Group

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Abstract:
The prime coprime graph of integers modulo n is an ordered pair consisting of a set of vertices (the integers modulo n) and a set of edges. Two distinct vertices are said to be adjacent if the greatest common divisor (gcd) of their orders equals 1 or a prime number. This article discusses the prime coprime graph of integers modulo n for $n = pq$, where $p < q$ are prime numbers. The results of the study include the degree characteristics of the vertices and the subgraphs formed. Additionally, the Sombor index of the graph is also determined.

Keywords: Prime Coprime Graph, Sombor Indices, Subgraph

1. Introduction

A graph is a mathematical structure used to model relationships between objects. Formally, a graph $G = (V, E)$ consists of a set of vertices V and edges E, where each edge connects a pair of vertices [1]. Graphs have become an important subject of study across various disciplines, including computer science, chemistry, and biology, due to their ability to represent complex relationships [2].

An interesting class of graph is the prime coprime graph on the group of integers modulo n. In this graph, two distinct vertices $a, b \in V$, $a \neq b$, are adjacent if and only if the greatest common divisor (GCD) of their orders is either 1 or a prime[3]. Adhikari first introduced this concept in [3], where the general characteristics of such graphs in group were explored. Subsequently, In the article by [4], the Wiener index and characteristics of prime coprime graphs on the group of integers modulo $n = p^k$, where p is a prime and $k \geq 2$ is a positive integer, were studied. Furthermore, the Padmakar–Ivan (PI), Szeged [5] and Harmonic, Randic, Gutman [6] and Zagreb-based [7] indices of prime coprime graphs have also been investigated. Therefore, this research focuses on the prime coprime graph in the group of integers modulo $n = pq$, where $p < q$ are prime numbers. This paper investigates the degree characteristics of the vertices and subgraphs.

As the study of graphs progresses, various topological indices have been developed to analyze graph properties. A topological index is a numerical parameter that reflects the characteristics of a graph in terms of its vertex and edge structure. Numerous studies have explored these indices. One particularly popular index is the Sombor index, introduced by [8], which is based on the degrees of vertices. In [9], the lower and upper bounds of the Sombor index were analyzed, while [10] explored the Sombor index in graph transformations. However, despite its popularity, there has been no study on the Sombor index for prime coprime graphs. Therefore, this research investigates the Sombor index for the prime coprime graph of the group of integers modulo $n = pq$.

2. Preliminaries

The prime coprime graph on integers modulo n, denoted by $\Gamma_{\mathbb{Z}_n}$, has a vertex set consisting of all elements of the group $\mathbb{Z}_n$. Two distinct vertices $u \neq v \in V(\Gamma_{\mathbb{Z}_n})$ are adjacent if and only if the greatest common divisor of the orders of the two vertices is either 1 or a prime number, i.e., $(|u|, |v|) = 1$ or $(|u|, |v|) = p$ [3].

Suppose the group $\mathbb{Z}_6$ is given. The prime coprime graph of this group can be constructed as follows

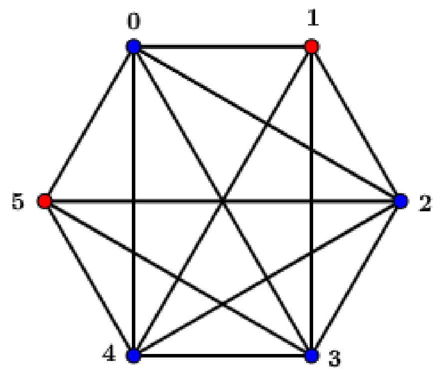


Figure 2.1: Prime coprime graph for $\mathbb{Z}_6$

The Sombor index is a degree-based graph index. It is defined as the square root of the sum of the squares of the degrees of adjacent vertices, $\sum_{uv \in E(\Gamma_{\mathbb{Z}_n})} \sqrt{\deg(u)^2 + \deg(v)^2}$, and is denoted by $SO(\Gamma_{\mathbb{Z}_n})$ [8]. The value of the Sombor index for the graph $\Gamma_{\mathbb{Z}_6}$ is as follows

$$
\begin{aligned}
SO(\Gamma_{\mathbb{Z}_6}) &= \sqrt{\deg(0)^2 + \deg(1)^2} + \sqrt{\deg(0)^2 + \deg(2)^2} + \sqrt{\deg(0)^2 + \deg(3)^2} + \sqrt{\deg(0)^2 + \deg(4)^2} \\
&\quad + \sqrt{\deg(0)^2 + \deg(5)^2} + \sqrt{\deg(1)^2 + \deg(2)^2} + \sqrt{\deg(1)^2 + \deg(3)^2} + \sqrt{\deg(1)^2 + \deg(4)^2} \\
&\quad + \sqrt{\deg(2)^2 + \deg(3)^2} + \sqrt{\deg(2)^2 + \deg(4)^2} + \sqrt{\deg(2)^2 + \deg(5)^2} + \sqrt{\deg(3)^2 + \deg(4)^2} \\
&\quad + \sqrt{\deg(3)^2 + \deg(5)^2} + \sqrt{\deg(4)^2 + \deg(5)^2} \\
&= \sqrt{5^2 + 4^2} + \sqrt{5^2 + 5^2} + \sqrt{5^2 + 5^2} + \sqrt{5^2 + 5^2} + \sqrt{5^2 + 4^2} + \sqrt{4^2 + 5^2} + \sqrt{4^2 + 5^2} \\
&\quad + \sqrt{4^2 + 5^2} + \sqrt{5^2 + 5^2} + \sqrt{5^2 + 5^2} + \sqrt{5^2 + 4^2} + \sqrt{5^2 + 5^2} + \sqrt{5^2 + 4^2} + \sqrt{5^2 + 4^2} \\
&= 6\sqrt{5^2 + 5^2} + 8\sqrt{5^2 + 4^2} \\
SO(\Gamma_{\mathbb{Z}_6}) &= 30\sqrt{2} + 8\sqrt{41}
\end{aligned}
$$

The next index is a modification of the Sombor index. The modification is made by subtracting 1 from the degree of each vertex. This index is called the Reduced Sombor Index, and its formula is $SO_{red}(\Gamma_{\mathbb{Z}_n}) = \sum_{uv \in E(\Gamma_{\mathbb{Z}_n})} \sqrt{(\deg(u) - 1)^2 + (\deg(v) - 1)^2}$ [11]. Using the calculations from the Sombor index, the value of the Reduced Sombor Index is obtained as follows:

$$
\begin{aligned}
SO_{red}\left(\Gamma_{\mathbb{Z}_6}\right) &= 6\sqrt{(5-1)^2+(5-1)^2}+8\sqrt{(5-1)^2+(4-1)^2} \\
&= 6\sqrt{32}+8\sqrt{25} \\
SO_{red}\left(\Gamma_{\mathbb{Z}_6}\right) &= 24\sqrt{2}+40
\end{aligned}
$$

A more general form of the Reduced Sombor Index is the Average Sombor Index, i.e $SO_{avg}\left(\Gamma_{\mathbb{Z}_n}\right) = \sum_{uv\in E(\Gamma_{\mathbb{Z}_n})}\sqrt{\left(\deg(u)-\frac{2m}{n}\right)^2+\left(\deg(v)-\frac{2m}{n}\right)^2}$ where m and n represent the number of edges and vertices, respectively [11]. Using the same calculation method as for the previous indices and given $m=14$ and $n=6$, the value of the Average Sombor Index is determined as follows:

$$
\begin{aligned}
SO_{avg}\left(\Gamma_{\mathbb{Z}_6}\right) &= 6\sqrt{\left(5-\frac{2\cdot 14}{6}\right)^2+\left(5-\frac{2\cdot 14}{6}\right)^2}+8\sqrt{\left(5-\frac{2\cdot 14}{6}\right)^2+\left(4-\frac{2\cdot 14}{6}\right)^2} \\
&= 6\sqrt{\left(\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)^2}+8\sqrt{\left(\frac{1}{3}\right)^2+\left(-\frac{2}{3}\right)^2} \\
SO_{avg}\left(\Gamma_{\mathbb{Z}_6}\right) &= 2\sqrt{2}+\frac{8}{3}\sqrt{5}
\end{aligned}
$$

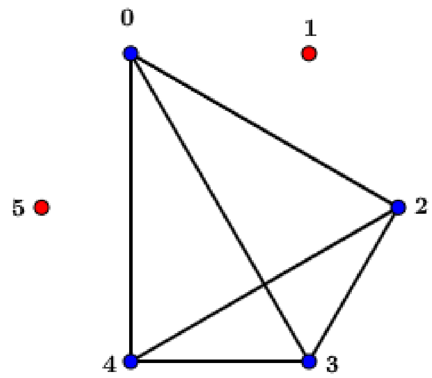
3. Results and Discussion

The adjacency between vertices in the graph $\Gamma_{\mathbb{Z}_6}$ can be analyzed based on the types of vertices. In Figure 2.1, the vertices can be grouped into two types, i.e $\{1,5\}$ and $\{0,2,3,4\}$. Furthermore, two vertices are adjacent if one of them is a multiple of 2 or 3. This can be observed as all vertices are adjacent to each other except for vertices 1 and 5, which are not adjacent.

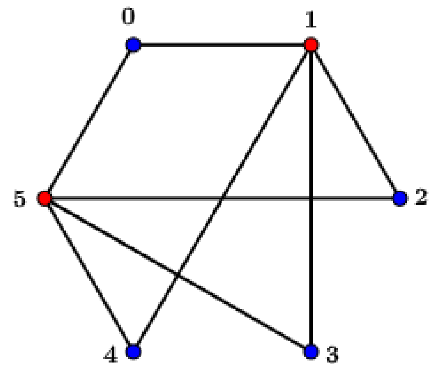
Theorem 3.1. *Given prime coprime graph $\Gamma_{\mathbb{Z}_n}$ with $n=pq$ and $p<q$ are primes, two distinct vertices $u,v\in V\left(\Gamma_{\mathbb{Z}_n}\right)$ are adjacent, $uv\in E\left(\Gamma_{\mathbb{Z}_n}\right)$, if $u\in\{0\}\cup\{p,2p,\ldots,(q-1)p\}$ or $v=\{0\}\cup\{q,2q,\ldots,(p-1)q\}$.*

Proof. Given $n=pq$ and let $V\left(\Gamma_{\mathbb{Z}_n}\right)$ be the set of vertices in the graph $\Gamma_{\mathbb{Z}_n}$. Partition of the set $V\left(\Gamma_{\mathbb{Z}_n}\right)$ into four subsets $A=\{p,2p,\ldots,(q-1)p\}$, $B=\{q,2q,\ldots,(p-1)q\}$, $C=\{0\}$, and $D=V\left(\Gamma_{\mathbb{Z}_n}\right)\backslash\{A\cup B\cup C\}$. Since $n=pq$ the cardinalities of the sets are $|a|=q$, $|b|=p$, $|c|=1$, and $|d|=n$ for all $a\in A, b\in B, c\in C$, and $d\in D$. For any $u\neq v\in V\left(\Gamma_{\mathbb{Z}_n}\right)$, the following properties hold based on the greatest common divisor (gcd) are $(|a_1|,|a_2|)=q$, $(|a_1|,|b_1|)=1$, $(|a_1|,|c|)=1$, $(|a_1|,|d_1|)=q$, $(|b_1|,|b_2|)=p$, $(|b_1|,|c|)=1$, $(|b_1|,|d_1|)=p$, $(|c|,|d_1|)=1$, and $(|d_1|,|d_2|)=n$. By the definition of the prime coprime graph, all vertices are connected except for $d_1\neq d_2\in D$. Thus, it is true that $uv\in E\left(\Gamma_{\mathbb{Z}_n}\right)$ for $u\neq v\in V\left(\Gamma_{\mathbb{Z}_n}\right)$ if $u\in\{0\}\cup\{p,2p,\ldots,(q-1)p\}$ or $v=\{0\}\cup\{q,2q,\ldots,(p-1)q\}$. $\square$

Refer to Figure 2.1. If the vertex $d\in D$ (red) and all edges incident to d are removed from the graph $\Gamma_{\mathbb{Z}_6}$, a complete subgraph K_4 is formed. Furthermore, if the edges within the complete subgraph are removed from $\Gamma_{\mathbb{Z}_6}$, the resulting graph becomes a complete bipartite subgraph $K_{4,2}$. The subgraphs are shown in the figure below.



(a) Complete Subgraph K_4



(b) Complete Bipartite Subgraph $K_{4,2}$

Figure 3.2: Subgraph of graph $\Gamma_{\mathbb{Z}_6}$

From the illustration of the graph $\Gamma_{\mathbb{Z}_6}$ (Figure 2.1 and 3.2), it can be observed that the subgraphs of the graph $\Gamma_{\mathbb{Z}_n}$ with $n = pq$ for $p < q$ are primes, are complete subgraphs and complete bipartite subgraphs. This is further clarified in the following theorem.

Theorem 3.2. *Given prime coprime graph $\Gamma_{\mathbb{Z}_n}$ with $n = pq$ and $p < q$ are primes, two subgraphs are formed, i.e complete subgraph K_{p+q-1} and complete bipartite $K_{p+q-1,(p-1)(q-1)}$*

Proof. Based on Theorem 3.1, two distinct vertices $u \neq v$ in the vertex sets A, B, and C are adjacent. Furthermore, $|A| = q - 1$, $|B| = p - 1$, and $|C| = 1$. Therefore, the graph formed by the vertices in $A \cup B \cup C$ is complete graph. Moreover, the total number of vertices is $p + q - 1$. Hence, the subgraph formed is the complete subgraph K_{p+q-1}.

Next, if the edges in the complete subgraph are removed, this results in $uv \notin E(\Gamma_{\mathbb{Z}_n})$ for $u \neq v \in D$ and $xy \notin E(\Gamma_{\mathbb{Z}_n})$ for $x \neq y \in A \cup B \cup C$. However, based on Theorem 3.1, it follows that $ux \in E(\Gamma_{\mathbb{Z}_n})$ for $u \in D$ and $x \in A \cup B \cup C$. Consequently, a complete bipartite graph is formed between the vertex sets D and $A \cup B \cup C$. Furthermore, the number of vertices in $n(A \cup B \cup C) = p+q-1$ and $n(D) = pq - (p + q - 1) = (p - 1)(q - 1)$. Thus, the subgraph is the complete bipartite graph $K_{p+q-1,(p-1)(q-1)}$. □

Next, if we look at Figure 2.1, it can be seen that the degrees of the vertices consist of two types, i.e $\deg(0) = \deg(2) = \deg(4) = \deg(6) = 6$ and $\deg(1) = \deg(5) = 4$. This holds for any $n = pq$ with $p < q$ is prime. Therefore, the degrees of the vertices in the graph $\Gamma_{\mathbb{Z}_n}$ can be categorized as stated in the following theorem.

Theorem 3.3. *Given prime coprime graph $\Gamma_{\mathbb{Z}_n}$ with $n = pq$ and $p < q$ are prime, there are two types of vertex degrees, i.e*

$$\deg(u) = \begin{cases} pq - 1 & , \textit{for } u \in A \cup B \cup C \\ p + q - 1 & , \textit{for } u \in D \end{cases}$$

with $A = \{p, 2p, \ldots, (q-1)p\}$, $B = \{q, 2q, \ldots, (p-1)q\}$, $C = \{0\}$, and $D = V(\Gamma_{\mathbb{Z}_n}) \setminus \{A \cup B \cup C\}$

Proof. Given that the number of vertices in the graph $\Gamma_{\mathbb{Z}_n}$ is pq. Based on Theorem 3.1, any vertex $u \in A \cup B \cup C$ is adjacent to all other vertices v where $u \neq v$. Therefore, $\deg(u) = pq - 1$.

Next, since $u \in D$, it will be adjacent to all vertices $v \in A \cup B \cup C$ and $n(A \cup B \cup C) = p + q - 1$, we have $\deg(u) = p + q - 1$. □

The distances between the vertices of the graph $\Gamma_{\mathbb{Z}_6}$ in Figure 2.1 are $d(0, 1) = d(0, 2) = \ldots = d(0, 5) = 1$ and $d(1, 5) = 2$. From these distances, it can be concluded that the distance between any two distinct vertices is either 1 or 2. The following theorem explains the distance between two distinct vertices in the graph $\Gamma_{\mathbb{Z}_n}$.

Theorem 3.4. *Given prime coprime graph $\Gamma_{\mathbb{Z}_n}$ with $n = pq$ and $p < q$ are prime, the distance between two distinct vertices, $u \neq v$, in the graph $\Gamma_{\mathbb{Z}_n}$ is as follows*

1. $d(u, v) = 1$ *for* $u, v \in A \cup B \cup C$
2. $d(u, v) = 1$ *for* $u \in A \cup B \cup C$ *and* $v \in D$
3. $d(u, v) = 2$ *for* $u, v \in D$

with $A = \{p, 2p, \dots, (q-1)p\}$, $B = \{q, 2q, \dots, (p-1)q\}$, $C = \{0\}$, *and* $D = V(\Gamma_{\mathbb{Z}_n}) \setminus \{A \cup B \cup C\}$.

Proof. From the explanation in Theorem 3.3, it is clear that any two vertices $u, v \in A \cup B \cup C$ are adjacent. Therefore, $d(u, v) = 1$. Furthermore, a vertex $u \in A \cup B \cup C$ and $v \in D$ are also adjacent, so $d(u, v) = 1$. Additionally, any two vertices $u, v \in D$ are not adjacent. However, the vertices $u, v \in D$ are adjacent to the same vertex in $A \cup B \cup C$. Hence, $d(u, v) = 2$. □

Theorem 3.5. *Given prime coprime graph $\Gamma_{\mathbb{Z}_n}$ with $n = pq$ and $p < q$ are prime. Sombor Index of graph $\Gamma_{\mathbb{Z}_n}$ is*

$$SO(\Gamma_{\mathbb{Z}_n}) = \frac{1}{2}\sqrt{2} \cdot (p+q-1)(p+q-2)(pq-1) + (p-1)(q-1)(p+q-1)\sqrt{(pq-1)^2 + (p+q-1)^2}$$

.

Proof. Let $V(\Gamma_{\mathbb{Z}_n})$ be partitioned into the sets $A = \{p, 2p, \dots, (q-1)p\}$, $B = \{q, 2q, \dots, (p-1)q\}$, $C = \{0\}$, and $D = V(\Gamma_{\mathbb{Z}_n}) \setminus \{A \cup B \cup C\}$. Based on Theorem 3.3, there are two types of vertex degrees. Let $\deg(u_1) = \deg(u_2) = pq - 1$ and $\deg(v_1) = p + q - 1$, where any vertices $u_1, u_2 \in A \cup B \cup C$ and $v_1 \in D$. According to Theorem 3.1, $u_1 u_2, u_1 v_1 \in E(\Gamma_{\mathbb{Z}_n})$ for any $v_1 \in D$. The number of edges connecting u_1 and u_2 is $C_2^{n(A \cup B \cup C)}$, while the number of edges connecting u_1 and v_1 is $n(D) \cdot n(A \cup B \cup C)$. Because $n(A) = q - 1, n(B) = p - 1$, and $n(C) = 1$ then $n(A \cup B \cup C) = p + q - 1$. Moreover, $n(D) = n(V) - n(A \cup B \cup C) = pq - (p + q - 1) = (p - 1)(q - 1)$. Hence, this equals C_2^{p+q-1} and $(p - 1)(q - 1) \cdot (p + q - 1)$, respectively. Observe the following,

$$\begin{aligned}
SO(\Gamma_{\mathbb{Z}_n}) &= \sum_{uv \in E(\Gamma_{\mathbb{Z}_n})} \sqrt{\deg(u)^2 + \deg(v)^2} \\
&= \sum_{u_1 u_2 \in E(\Gamma_{\mathbb{Z}_n})} \sqrt{\deg(u_1)^2 + \deg(u_2)^2} + \sum_{u_1 v_1 \in E(\Gamma_{\mathbb{Z}_n})} \sqrt{\deg(u_1)^2 + \deg(v_1)^2} \\
&= C_2^{p+q-1}\sqrt{(pq-1)^2 + (pq-1)^2} + (p-1)(q-1)(p+q-1)\sqrt{(pq-1)^2 + (p+q-1)^2} \\
&= \frac{(p+q-1)!}{(p+q-3)! \cdot 2!} \cdot (pq-1)\sqrt{2} + (p-1)(q-1)(p+q-1)\sqrt{(pq-1)^2 + (p+q-1)^2} \\
SO(\Gamma_{\mathbb{Z}_n}) &= \frac{1}{2}\sqrt{2} \cdot (p+q-1)(p+q-2)(pq-1) + (p-1)(q-1)(p+q-1)\sqrt{(pq-1)^2 + (p+q-1)^2}.
\end{aligned}$$

□

Theorem 3.6. *Given prime coprime graph $\Gamma_{\mathbb{Z}_n}$ with $n = pq$ and $p < q$ are prime. The Reduced Sombor Index of graph $\Gamma_{\mathbb{Z}_n}$ is follow*

$$SO_{red}(\Gamma_{\mathbb{Z}_n}) = \frac{1}{2}\sqrt{2} \cdot (p+q-1)(p+q-2)(pq-2) + (p-1)(q-1)(p+q-1)\sqrt{(pq-2)^2 + (p+q-2)^2}$$

.

Proof. With the same assumptions as in the proof of Theorem 3.5, the Reduced Sombor Index of the prime coprime graph 3.5 is derived as follows

$$
\begin{aligned}
SO_{red}\left(\Gamma_{\mathbb{Z}_n}\right) &= \sum_{uv\in E(\Gamma_{\mathbb{Z}_n})} \sqrt{(\deg(u)-1)^2+(\deg(v)-1)^2} \\
&= \sum_{u_1u_2\in E(\Gamma_{\mathbb{Z}_n})} \sqrt{(\deg(u_1)-1)^2+(\deg(u_2)-1)^2} + \sum_{u_1v_1\in E(\Gamma_{\mathbb{Z}_n})} \sqrt{(\deg(u_1)-1)^2+(\deg(v_1)-1)^2} \\
&= C_2^{p+q-1}\sqrt{(pq-2)^2+(pq-2)^2} + (p-1)(q-1)(p+q-1)\sqrt{(pq-2)^2+(p+q-2)^2} \\
SO_{red}\left(\Gamma_{\mathbb{Z}_n}\right) &= \frac{1}{2}\sqrt{2}\cdot(p+q-1)(p+q-2)(pq-2) + (p-1)(q-1)(p+q-1)\sqrt{(pq-2)^2+(p+q-2)^2}.
\end{aligned}
$$

□

Theorem 3.7. *Given prime coprime graph $\Gamma_{\mathbb{Z}_n}$ with $n = pq$ and $p < q$ are prime. The Average Sombor Index of graph $\Gamma_{\mathbb{Z}_n}$ is follow*

$$
\begin{aligned}
SO_{avg}\left(\Gamma_{\mathbb{Z}_n}\right) &= \frac{1}{2}(p+q-1)(p+q-2)\cdot\left(pq-1-\frac{(p+q-1)(2pq-p-q)}{pq}\right)\sqrt{2} \\
&+ (p-1)(q-1)(p+q-1) \\
&\sqrt{\left(pq-1-\frac{(p+q-1)(2pq-p-q)}{pq}\right)^2+\left(p+q-1-\frac{(p+q-1)(2pq-p-q)}{pq}\right)^2}.
\end{aligned}
$$

Proof. The total number of vertices in the graph $\Gamma_{\mathbb{Z}_n}$ is $n = pq$ and the total number of edges is

$$
\begin{aligned}
m &= C_2^{p+q-1} + (p-1)(q-1)(p+q-1) \\
&= \frac{1}{2}(p+q-1)(p+q-2) + (p-1)(q-1)(p+q-1) \\
&= (p+q-1)\left(\frac{1}{2}(p+q-2)+(p-1)(q-1)\right) \\
&= \frac{1}{2}(p+q-1)(2pq-p-q).
\end{aligned}
$$

Using the same reasoning as in Theorem 3.5, the total number of edges in prime coprime graph $\Gamma_{\mathbb{Z}_n}$ is obtained as

$$
\begin{aligned}
SO_{avg}\left(\Gamma_{\mathbb{Z}_n}\right) &= \sum_{uv\in E(\Gamma_{\mathbb{Z}_n})} \sqrt{\left(\deg(u)-\frac{2m}{n}\right)^2+\left(\deg(v)-\frac{2m}{n}\right)^2} \\
&= \sum_{u_1u_2\in E(\Gamma_{\mathbb{Z}_n})} \sqrt{\left(\deg(u_1)-\frac{2m}{n}\right)^2+\left(\deg(u_2)-\frac{2m}{n}\right)^2} \\
&+ \sum_{u_1v_1\in E(\Gamma_{\mathbb{Z}_n})} \sqrt{\left(\deg(u_1)-\frac{2m}{n}\right)^2+\left(\deg(v_1)-\frac{2m}{n}\right)^2} \\
SO_{avg}\left(\Gamma_{\mathbb{Z}_n}\right) &= \frac{1}{2}(p+q-1)(p+q-2)\cdot\left(pq-1-\frac{(p+q-1)(2pq-p-q)}{pq}\right)\sqrt{2} \\
&+ (p-1)(q-1)(p+q-1) \\
&\sqrt{\left(pq-1-\frac{(p+q-1)(2pq-p-q)}{pq}\right)^2+\left(p+q-1-\frac{(p+q-1)(2pq-p-q)}{pq}\right)^2}.
\end{aligned}
$$

□

4. Conclusions

The prime coprime graph on the group of integers modulo n with $n = pq$ and $p < q$ are primes, consists of two primary subgraphs, i.e a complete subgraph K_{p+q-1} and complete bipartite subgraph $K_{p+q-1,(p-1)(q-1)}$. The vertex degrees in this graph are of two types, $pq-1$ and $p+q-1$, where $p < q$ is prime. Additionally, three Sombor indices can be derived for this graph.

1. The Sombor Index

$$SO\left(\Gamma_{\mathbb{Z}_n}\right) = \frac{1}{2}\sqrt{2}\cdot(p+q-1)(p+q-2)(pq-1) + (p-1)(q-1)(p+q-1)\sqrt{(pq-1)^2+(p+q-1)^2}$$

.

2. The Reduced Sombor Index

$$SO_{red}\left(\Gamma_{\mathbb{Z}_n}\right) = \frac{1}{2}\sqrt{2}\cdot(p+q-1)(p+q-2)(pq-2) + (p-1)(q-1)(p+q-1)\sqrt{(pq-2)^2+(p+q-2)^2}$$

.

3. The Average Sombor Index

$$\begin{aligned} SO_{avg}\left(\Gamma_{\mathbb{Z}_n}\right) = {} & \frac{1}{2}(p+q-1)(p+q-2)\cdot\left(pq-1-\frac{(p+q-1)(2pq-p-q)}{pq}\right)\sqrt{2} \\ & + (p-1)(q-1)(p+q-1) \\ & \sqrt{\left(pq-1-\frac{(p+q-1)(2pq-p-q)}{pq}\right)^2+\left(p+q-1-\frac{(p+q-1)(2pq-p-q)}{pq}\right)^2}. \end{aligned}$$

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