

RESEARCH ARTICLE

A Novel Approach to Topological Indices of the Identity Graph Associated with the Finite Group

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Abstract:

The combination of graphs and algebra has become a widely discussed topic in research within the fields of algebra and combinatorics. Research on group representations on graphs and topological indices has been extensively conducted, one such example is on the identity graph. An identity graph of a group G which is an ordered pair $V(G)$ and $E(G)$, where all elements of G serve as vertices, and two vertices $x, y \in G$ are adjacent if and only if $x * y = e$. This study proposes an alternative approach to calculating topological indices in the identity graph of the multiplicative group of integers modulo n . As for the stages, they are divided into several steps, namely: first, find the graph form based on the definition of the identity graph of a group. Next, based on the graph's illustration, determine the number of vertices and edges. Finally, substitute them into the formula given in the theorem.

Keywords: Graph theory, representation group, identity graph, alternative approach, topological indices

1. Introduction

Graph theory is utilized to analyze network structures in various fields, including computer science and discrete mathematics. The combination of graphs and algebra has become a widely discussed topic in research within the fields of algebra and combinatorics. Group representation on graphs is often used to study the symmetry and automorphism of a mathematical structure. In recent years, research on group representation in graphs has been growing and gaining significant attention, some of them are The coprime graph [1, 2], the non-coprime graph [3], the relatively coprime graph [4], the intersection graph [5], the power graph [6–8], and the identity graph [9] are some of the key topics in recent studies. Furthermore, in the book, a definition of graph representation concerning the modulo n group was introduced. Let Γ_G be the identity graph of a group G , which is an ordered pair $V(G)$ and $E(G)$, where all elements of G serve as vertices, and two vertices $x, y \in G$ are adjacent if and only if $x * y = e$

In 2023, a study examined the topological indices of the power graph of the dihedral group. The results of this research included the first Zagreb index, the Wiener index, and the Gutman index [10]. One year later, introduced a new approach to computing the first Zagreb index, the second Zagreb index, and the Gutman index in the case of the zero-divisor graph of a commutative ring [11]. And other research about topological indices [12, 13]. However, in the case of the zero-divisor graph of a commutative ring, additional considerations were required. Therefore, this study presents a new approach to calculating topological indices in the identity graph of the multiplicative group of integers modulo n .

2. Method

The method employed in this research is a literature review of various studies conducted by previous researchers, followed by a case study of several forms of graph representations of algebraic structures and the calculation of their topological indices. Subsequently, several conjectures were obtained and then proven, resulting in a generalization. If proven, this generalization will become a theorem, whereas if unproven, a reanalysis will be conducted on the conjectures obtained from the case study.

3. Results and Discussion

Before discussing the results, the following materials were used to support this research. This research focuses on a new approach to calculating topological indices based on distances in the identity graph of integer modulo group. The research yielded calculations for the Wiener index, Hyper-Wiener index, and Harary index, utilizing this new approach to calculate topological indices. Here are some general definitions of the topological indices to be investigated.

Definition 3.1. [9] An identity graph of a group of integers modulo n , denoted $\Gamma_{\mathbb{Z}_n}$, is an order paired $V(\Gamma_{\mathbb{Z}_n})$ and $E(\Gamma_{\mathbb{Z}_n})$, where all elements of group $\mathbb{Z}_n$ serve as vertices, and two vertices $x, y \in \Gamma_{\mathbb{Z}_n}$ are adjacent if and only if $x * y = e$, and all of $V(\Gamma_{\mathbb{Z}_n})$ without e are adjacent to e .

Definition 3.2. [14] Wiener Index

Let Γ be a connected graph. The Wiener index of Γ is the sum of the half of the distances between every unordered pair of vertices of Γ , written as,

$$W(\Gamma) = \sum_{u,v \in V(\Gamma)} d(u, v)$$

where $d(u, v)$ is the distances of unordered pair of vertex u and v .

Definition 3.3. [15] Hyper-Wiener Index

Let Γ be a connected graph. The hyper-Wiener index of Γ , denoted by $WW(\Gamma)$, is defined as

$$WW(\Gamma) = \frac{1}{2} \left(\sum_{u,v \in V(\Gamma)} d(u, v) + \sum_{u,v \in V(\Gamma)} d(u, v)^2 \right)$$

where $d(u, v)$ is the distances of unordered pair of vertex u and v .

Definition 3.4. [15] Harary index

Let Γ be a connected graph. The Harary index of Γ , denoted by $H(\Gamma)$, is defined as

$$H(\Gamma) = \sum_{u,v \in V(\Gamma)} \frac{1}{d(u, v)}$$

where $d(u, v)$ is the distances of unordered pair of vertex u and v .

According to the study by [11], a new approach to calculating topological indices was established, focusing exclusively on vertex and edge cardinalities. The following lemmas were derived:

Lemma 3.1. [11] *Let Γ be a simple connected graph with $\text{diam}(\Gamma) \leq 2$, then the Wiener index of Γ is $|V(\Gamma)|(|V(\Gamma)| - 1) - |E(\Gamma)|$.*

Proof. Since $\text{diam}(\Gamma) \leq 2$, the number of unordered pairs of vertices in Γ that have distance 2 is,

$$\binom{|V(\Gamma)|}{2} - |E(\Gamma)|.$$

Hence, the Wiener index of Γ is,

$$\begin{aligned} W(\Gamma) &= |E(\Gamma)| + 2 \left(\binom{|V(\Gamma)|}{2} - |E(\Gamma)| \right) \\ &= |V(\Gamma)|(|V(\Gamma)| - 1) - |E(\Gamma)|. \end{aligned}$$

□

Lemma 3.2. [11] *Let Γ be a simple connected graph with $\text{diam}(\Gamma) \leq 2$, then the hyper-Wiener index of Γ is $\frac{3}{2}|V(\Gamma)|(|V(\Gamma)| - 1) - 2|E(\Gamma)|$.*

Proof. Since $\text{diam}(\Gamma) \leq 2$, the number of unordered pairs of vertices in Γ that have distance 2 is,

$$\binom{|V(\Gamma)|}{2} - |E(\Gamma)|.$$

Hence, the hyper-Wiener index of Γ is,

$$\begin{aligned} WW(\Gamma) &= \frac{1}{2} \left[|E(\Gamma)| + 2 \left(\binom{|V(\Gamma)|}{2} - |E(\Gamma)| \right) + |E(\Gamma)| + 4 \left(\binom{|V(\Gamma)|}{2} - |E(\Gamma)| \right) \right] \\ &= \frac{3}{2} |V(\Gamma)|(|V(\Gamma)| - 1) - 2|E(\Gamma)|. \end{aligned}$$

□

Lemma 3.3. [11] *Let Γ be a simple connected graph with $\text{diam}(\Gamma) \leq 2$, then the Harary index of Γ is $\frac{1}{4}|V(\Gamma)|(|V(\Gamma)| - 1) + \frac{1}{2}|E(\Gamma)|$.*

Proof. Since $\text{diam}(\Gamma) \leq 2$, the number of unordered pairs of vertices in Γ that have distance 2 is,

$$\binom{|V(\Gamma)|}{2} - |E(\Gamma)|.$$

Hence, the Harary index of Γ is,

$$\begin{aligned} H(\Gamma) &= |E(\Gamma)| + \frac{1}{2} \left(\binom{|V(\Gamma)|}{2} - |E(\Gamma)| \right) \\ &= \frac{1}{4} |V(\Gamma)|(|V(\Gamma)| - 1) + \frac{1}{2} |E(\Gamma)|. \end{aligned}$$

□

Aligned with the research objective of calculating topological indices for identity graph of integer modulo group using new approaches (specifically Lemma 3.1, Lemma 3.2, and Lemma 3.3 presented above), our analysis of identity graph of integer modulo group with order n (where n is a positive integer) will be divided into two cases: (1) when n is odd and (2) when n is even. The following research findings are presented:

Theorem 3.1. Edges

Let $\Gamma_{\mathbb{Z}_n}$ be identity graph of integer modulo n . Then the total edges of $\Gamma_{\mathbb{Z}_n}$ is

$$|E(\Gamma_{\mathbb{Z}_n})| = \begin{cases} \frac{3(n-1)}{2} & , n \text{ is odd number} \\ \frac{3n-4}{2} & , n \text{ is even} \end{cases}$$

Proof. To prove this theorem, we use handshaking lemma and Theorem about degree in [9].

If n is odd number then the result is as follows

$$\begin{aligned} 2|E(\Gamma_{\mathbb{Z}_n})| &= \sum_{u \in \mathbb{Z}_n} \deg(u) \\ &= \deg(0) + \sum_{u \in \mathbb{Z}_n - \{0\}} \deg(u) \\ &= (n-1) + (n-1) \bullet 2 \\ &= 3(n-1) \\ 2|E(\Gamma_{\mathbb{Z}_n})| &= 3(n-1) \\ |E(\Gamma_{\mathbb{Z}_n})| &= \frac{3(n-1)}{2} \end{aligned}$$

If n is even number then we get

$$\begin{aligned} 2|E(\Gamma_{\mathbb{Z}_n})| &= \sum_{u \in \mathbb{Z}_n} \deg(u) \\ &= \deg(0) + \deg\left(\frac{n}{2}\right) + \sum_{u \in \mathbb{Z}_n - \{0, \frac{n}{2}\}} \deg(u) \\ &= n-1 + 1 + (n-2) \bullet 2 \\ &= 3n-4 \\ 2|E(\Gamma_{\mathbb{Z}_n})| &= 3n-4 \\ |E(\Gamma_{\mathbb{Z}_n})| &= \frac{3n-4}{2} \end{aligned}$$

□

Theorem 3.2. Wiener Index

Let $\Gamma_{\mathbb{Z}_n}$ be identity graph of integer modulo n . Then the Wiener index of $\Gamma_{\mathbb{Z}_n}$ is

$$W(\Gamma_{\mathbb{Z}_n}) = \begin{cases} \frac{2n^2-5n+3}{2} & , n \text{ is odd number} \\ \frac{2n^2-5n+4}{2} & , n \text{ is even} \end{cases}$$

Proof. If n is odd number then we obtain

$$\begin{aligned} W(\Gamma_{\mathbb{Z}_n}) &= |V(\Gamma_{\mathbb{Z}_n})| (|V(\Gamma_{\mathbb{Z}_n})| - 1) - E(\Gamma_{\mathbb{Z}_n}) \\ &= n(n-1) - \frac{3(n-1)}{2} \\ &= \frac{2n^2-5n+3}{2} \end{aligned}$$

If n is even number then the result is as follows

$$\begin{aligned} W(\Gamma_{\mathbb{Z}_n}) &= |V(\Gamma_{\mathbb{Z}_n})| (|V(\Gamma_{\mathbb{Z}_n})| - 1) - E(\Gamma_{\mathbb{Z}_n}) \\ &= n(n-1) - \frac{3n-4}{2} \\ &= \frac{2n^2-5n+4}{2} \end{aligned}$$

□

Theorem 3.3. *Hyper-Wiener Index*

Let $\Gamma_{\mathbb{Z}_n}$ be identity graph of integer modulo n . Then the Hyper-Wiener index of $\Gamma_{\mathbb{Z}_n}$ is

$$WW(\Gamma_{\mathbb{Z}_n}) = \begin{cases} \frac{3}{2}(n^2 - 3n + 2) & , n \text{ is odd number} \\ \frac{3n^2 - 9n + 8}{2} & , n \text{ is even} \end{cases}$$

Proof. If n is odd number then we have

$$\begin{aligned} WW(\Gamma_{\mathbb{Z}_n}) &= \frac{3}{2} |V(\Gamma_{\mathbb{Z}_n})| (|V(\Gamma_{\mathbb{Z}_n})| - 1) - 2 |E(\Gamma_{\mathbb{Z}_n})| \\ &= \frac{3}{2} (n)(n - 1) - 2 \left(\frac{3(n - 1)}{2} \right) \\ &= \frac{3n^2 - 9n + 6}{2} \\ &= \frac{3}{2} (n^2 - 3n + 2) \end{aligned}$$

If n is even number then the value is given by

$$\begin{aligned} WW(\Gamma_{\mathbb{Z}_n}) &= \frac{3}{2} |V(\Gamma_{\mathbb{Z}_n})| (|V(\Gamma_{\mathbb{Z}_n})| - 1) - 2 |E(\Gamma_{\mathbb{Z}_n})| \\ &= \frac{3}{2} (n)(n - 1) - 2 \left(\frac{3n - 4}{2} \right) \\ &= \frac{3n^2 - 9n + 8}{2} \end{aligned}$$

□

Theorem 3.4. *Harary Index*

Let $\Gamma_{\mathbb{Z}_n}$ be identity graph of integer modulo n . Then the Harary index of $\Gamma_{\mathbb{Z}_n}$ is

$$H(\Gamma_{\mathbb{Z}_n}) = \begin{cases} \frac{n^2 + 2n - 3}{4} & , n \text{ is odd number} \\ \frac{n^2 + 2n - 4}{4} & , n \text{ is even} \end{cases}$$

Proof. If n is odd number then the formula becomes

$$\begin{aligned} H(\Gamma_{\mathbb{Z}_n}) &= \frac{1}{4} |V(\Gamma_{\mathbb{Z}_n})| (|V(\Gamma_{\mathbb{Z}_n})| - 1) + \frac{1}{2} |E(\Gamma_{\mathbb{Z}_n})| \\ &= \frac{1}{4} (n)(n - 1) + \frac{1}{2} \left(\frac{3(n - 1)}{2} \right) \\ &= \frac{n^2 + 2n - 3}{4} \end{aligned}$$

If n is even number then the value is given by

$$\begin{aligned} H(\Gamma_{\mathbb{Z}_n}) &= \frac{1}{4} |V(\Gamma_{\mathbb{Z}_n})| (|V(\Gamma_{\mathbb{Z}_n})| - 1) + \frac{1}{2} |E(\Gamma_{\mathbb{Z}_n})| \\ &= \frac{1}{4} (n)(n - 1) + \frac{1}{2} \left(\frac{3n - 4}{2} \right) \\ &= \frac{n^2 + 2n - 4}{4} \end{aligned}$$

□

Example 3.1. Let $\Gamma_{\mathbb{Z}_n}$ be an identity graph of order $n = 7$. First, determine the vertex set $V(\Gamma_{\mathbb{Z}_n}) = \{e, 1, 2, 3, 4, 5, 6\}$, and the edge set $E(\Gamma_{\mathbb{Z}_n}) = \{(e, 1), (e, 2), (e, 3), (e, 4), (e, 5), (e, 6), (1, 6), (2, 5), (3, 4)\}$. Next, construct the graph according to the definition of an identity graph.. Hence, the graph can be represented as follows:

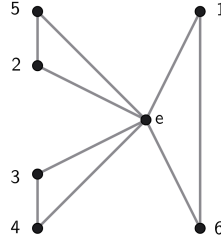


Figure 3.1: $\Gamma_{\mathbb{Z}_7}$

As can be easily seen in Figure 3.1, the distances between any two vertices are given below: $d(e, 1) = d(e, 2) = \dots = d(e, 6) = d(3, 4) = d(2, 5) = d(1, 6) = 1$ and $d(1, 2) = d(1, 3) = \dots = d(4, 5) = d(4, 6) = d(5, 6) = 2$.

By Definition 3.2, Wiener Index of $\Gamma_{\mathbb{Z}_7}$, written as,

$$\begin{aligned} W(\Gamma) &= \sum_{u,v \in V(\Gamma)} d(u, v) \\ &= d(e, 1) + d(e, 2) + \dots + d(e, 6) + d(3, 4) + d(2, 5) + d(1, 6) + d(1, 2) + \dots + d(5, 6) \\ &= \underbrace{1 + 1 + \dots + 1}_{9x} + \underbrace{2 + 2 + \dots + 2}_{12x} \\ &= 9(1) + 12(2) \\ &= 33 \end{aligned}$$

By Theorem 3.2 a new approach Wiener Index, because $n = 7(\text{odd})$, then the Wiener index of $\Gamma_{\mathbb{Z}_7}$ is

$$\begin{aligned} W(\Gamma_{\mathbb{Z}_7}) &= \frac{2(7)^2 - 5(7) + 3}{2} \\ &= \frac{66}{2} \\ &= 33 \end{aligned}$$

Since the results match, the new approach clearly provides a more streamlined calculation.

4. Conclusion

This study presents a new approach to calculating the Wiener index, Hyper-Wiener index, and Harary index of the identity graph of integer modulo group with order n , where $n \in \mathbb{Z}$. For further development, similar investigations should be conducted on graphs derived from other algebraic structures by applying this new calculating approach topological indices.

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