J. DIFERENSIAL/e-ISSN:2775-9644
Vol.8(2),2026, 111–119

doi:https://doi.org/10.35508/jd.v8i2.27486

RESEARCH ARTICLE

A Hybrid Semi-Analytical Technique for the Homogeneous Space Fractional Damped Wave Equation with Gaussian White Noise

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Received: 14 February 2026; Revised: 4 March 2026; Accepted: 10 April 2026; Published: 14 May 2026.

Abstract:
This paper addresses the severely ill-posed final value problem for the homogeneous space fractional damped wave equation subject to Gaussian white noise. Unlike the well-posed forward problem, recovering the initial state from noisy final data is unstable, as high-frequency noise components are amplified exponentially. We propose the Laplace-Residual Power Series Method (LRPSM), a semi-analytical iterative technique, to solve this problem. By transforming the backward problem into a time-reversed initial value problem, we construct a series solution in the Laplace domain. We provide a rigorous theorem and proof regarding the convergence of the method for exact data and discuss its regularizing properties via series truncation for noisy data. A numerical example is presented to illustrate the accuracy and stability of the proposed method compared to standard Fourier truncation techniques.

Keywords: Space fractional damped wave equation; Final value problem; Laplace-Residual Power Series Method; Gaussian white noise; Ill-posedness.

1. Introduction

The study of damped wave equations is fundamental in mathematical physics and engineering, modeling phenomena where wave propagation is accompanied by energy dissipation, such as in viscoelastic materials, telegraphy, and relativistic quantum mechanics [1, 2]. In recent decades, the incorporation of fractional calculus has refined these models. The space fractional damped wave equation, which replaces the standard Laplacian with the fractional Laplacian operator $(-\Delta)\gamma$, allows for the description of non-local interactions and anomalous diffusion in heterogeneous media [3, 4].

In this work, we consider the Final Value Problem (FVP) for the homogeneous space fractional damped wave equation. While the direct problem (finding the state of the system at time $t > 0$ given the initial state at $(t = 0)$ is well-posed, the inverse problem of reconstructing the past state given the final data at time T is severely ill-posed in the sense of Hadamard [5]. The primary difficulty arises from the fact that the solution operators for dissipative wave equations are smoothing operators in

the forward time direction. Consequently, in the backward direction, high-frequency components grow exponentially. When the final data is contaminated by random errors specifically Gaussian white noise these errors are amplified catastrophically, rendering standard numerical schemes unstable [6, 7].
The problem is formulated as follows: Find the function $u(x, t)$ satisfying:

$$u_{tt} + (-\Delta)^{\gamma} u + u_t + (-\Delta)^{\gamma} u_t = 0, \quad (x, t) \in (0, 1) \times [0, T], \tag{1.1}$$

subject to the boundary conditions $u(0, t) = u(1, t) = 0$, and the noisy final conditions:

$$u(x, T) = g_{\epsilon}(x), \quad u_t(x, T) = h_{\epsilon}(x). \tag{1.2}$$

Here, g_{ϵ} and h_{ϵ} represent the measured final position and velocity, respectively, perturbed by Gaussian white noise with intensity ϵ.

To address the ill-posedness, regularization methods such as Tikhonov regularization, Quasi-Boundary Value Methods (QBVM), and Fourier truncation are typically employed [8–10]. Recently, Huy [11] utilized a Fourier truncation method for this specific equation. However, spectral cut-off methods can be sensitive to parameter selection and computationally expensive for non-linear extensions.

In this paper, we propose the LRPSM. This method combines the Laplace transform with the Residual Power Series Method (RPSM) [12–14]. By applying a time-reversal transformation, we convert the final value problem into an initial value problem. The LRPSM then constructs the solution as a rapidly convergent power series in the Laplace domain. The truncation of this series acts as a natural regularization filter, stabilizing the solution against the high-frequency noise inherent in the input data.

2. Basic Concepts

Let $H = L^2(0, 1)$ be the Hilbert space with inner product $\langle u, v\rangle = \int_0^1 u(x)v(x)\,dx$. The orthonormal basis functions are $\psi n(x) = \sqrt{2}\sin(n\pi x)$ with eigenvalues $\lambda_n = (n\pi)^2$.

Definition 2.1. For $\gamma \in (0, 1)$, the fractional Laplacian $(-\Delta)^{\gamma}$ is defined by spectral decomposition:

$$(-\Delta)^{\gamma} v(x) = \sum_{n=1}^{\infty} \lambda_n^{\gamma} \langle v, \psi_n\rangle \psi_n(x). \tag{2.3}$$

The domain $D((-\Delta)^{\gamma})$ consists of functions $v \in L^2(0, 1)$ such that $\sum_{n=1}^{\infty} \lambda_n^{2\gamma} |\langle v, \psi_n\rangle|^2 < \infty$.

Definition 2.2. Let (Ω, F, P) be a probability space. The noisy data is modeled as:

$$g_{\epsilon}(x) = g_{ex}(x) + \epsilon \dot{W}(x), \quad h_{\epsilon}(x) = h_{ex}(x) + \epsilon \dot{W}(x), \tag{2.4}$$

where g_{ex}, h_{ex} are the exact data and $\dot{W}(x)$ denotes spatial Gaussian white noise [15].

Definition 2.3. The Caputo fractional derivative $D_t^{\alpha} w(t, x)$ of α-order is defined in [16] as

$$D_t^{\alpha} w(t, x) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{1}{(t-p)^{n-\alpha-1}} \frac{\partial^n w(p, x)}{\partial p^{\alpha}}\, dp, \quad (n - 1 < \alpha \le n), \tag{2.5}$$

if $\alpha = n \in N$, then we can write as:

$$D_t^{\alpha} w(t, x) = \frac{D^{\alpha} w(t, x)}{\partial t^{\alpha}} = \frac{D^{n} w(t, x)}{\partial t^{n}}.$$

3. Laplace-Residual Power Series Method

To apply LRPSM, we first perform a time-reversal transformation. Let $\tau = T - t$. Define $v(x, \tau) = u(x, T - \tau)$. The problem (1.1) transforms into:

$$v_{\tau\tau} + (-\Delta)^\gamma v + v_\tau + (-\Delta)^\gamma v_\tau = 0, \quad \tau \in [0, T], \tag{3.6}$$

with initial conditions (derived from the noisy final data):

$$v(x, 0) = g_\epsilon(x), \quad v_\tau(x, 0) = -h_\epsilon(x). \tag{3.7}$$

Applying the Laplace transform $\mathcal{L}[\cdot]$ with respect to τ to equation (3.6), and letting $V(x, s) = \mathcal{L}[v(x, \tau)]$:

$$(s^2 - s)V(x, s) - (s - 1)v(x, 0) - v_\tau(x, 0) + (1 - s)(-\Delta)^\gamma V(x, s) + (-\Delta)^\gamma v(x, 0) = 0. \tag{3.8}$$

The LRPSM assumes a series solution of the form:

$$V(x, s) = \sum_{k=0}^{\infty} \frac{f_k(x)}{s^{k+1}}. \tag{3.9}$$

Using the initial value theorem, $f_0(x) = v(x, 0) = g_\epsilon(x)$ and $f_1(x) = v_\tau(x, 0) = h(x)$. We define the N-th truncated solution $V_N(x, s) = \sum_{k=0}^{N} \frac{f_k(x)}{s^{k+1}}$ and the Laplace-Residual function:

$$\text{Res}(x, s) = (s^2 - s)V(x, s) + (1 - s)(-\Delta)^\gamma V(x, s) - (s - 1)f_1 - f_0 + (-\Delta)^\gamma f_0. \tag{3.10}$$

To determine the coefficients f_k for $k \geq 2$, we impose the limit condition:

$$\lim_{s \to \infty} s^{k+1}\, \text{Res}_k(x, s) = 0. \tag{3.11}$$

Substituting the series into (3.10) and applying (3.11) yields the recurrence relation:

$$f_{k+2}(x) = f_{k+1}(x) + (-\Delta)^\gamma f_{k+1}(x) - (-\Delta)^\gamma f_k(x). \tag{3.12}$$

Finally, applying the inverse Laplace transform, the approximate solution in the original time variable t is:

$$u_N(x, t) = \sum_{k=0}^{N} f_k(x) \frac{(T - t)^k}{k!}. \tag{3.13}$$

4. Convergence Analysis

In this section, we establish the convergence of the LRPSM solution to the exact solution for the noise-free case ($\epsilon = 0$). This theoretical foundation justifies the method's use. For the noisy case, the integer N acts as the regularization parameter.

Theorem 4.1. *Let $u_{ex}(x, t)$ be the unique exact solution to the problem (1.1)–(1.2) with exact data $g_{ex}, h_{ex} \in D((-\Delta)^\infty)$.*

Let $u_N(x, t)$ be the N-th order approximate solution obtained via LRPSM as defined in equation (3.13). Then, the sequence of approximate solutions $\{u_N(x, t)\}_N^\infty = 0$ uniformly convergent to the exact solution $u_{ex}(x, t)$ on the domain $(x, t) \in [0, 1] \times [0, T]$.

Proof. The proof relies on the equivalence between the LRPSM coefficients and the Taylor series coefficients of the exact solution.
Step 1: Identifying the LRPSM Coefficients.

The recurrence relation derived from the LRPSM in equation (3.11) is:

$$f_{k+2} = f_{k+1} + (-\Delta)^{\gamma}(f_{k+1} - f_k), \quad k \geq 0, \tag{4.14}$$

with $f_0 = g_{ex}$ and $f_1 = h_{ex}$.
Consider the time-reversed equation

$$v_{\tau\tau} - v_{\tau} + (-\Delta)^{\gamma}(v - v_{\tau}) = 0.$$

By differentiating this equation k times with respect to τ and evaluating at $\tau = 0$, we observe that the coefficients of the Taylor expansion of the exact solution, denoted by $c_k = \frac{\partial^k v}{\partial \tau^k}(x, 0)$, satisfy exactly the same recurrence relation as the LRPSM relation. Since the initial conditions match ($c_0 = f_0,\ c_1 = f_1$), it implies

$$f_k(x) = \frac{\partial^k v}{\partial \tau^k}(x, 0), \quad \text{for all } k \geq 0.$$

Step 2: Aalyticity of the Exact Solution.

Since the operator $A = I + (-\Delta)^{\gamma}$ is a self-adjoint, positive-definite operator on H with discrete spectrum $\mu_n = 1 + (n\pi)^{2\gamma}$, the solution $v(x, \tau)$ can be represented via spectral expansion.

$$v(x, \tau) = \sum_{n=1}^{\infty} \left(A_n e^{\alpha_n \tau} + B_n e^{\beta_n \tau} \right) \psi_n(x), \tag{4.15}$$

where α_n, β_n are roots of the characteristic equation $r^2 + (\mu_n - 1)r + \cdots$. Crucially, for any fixed spatial mode n, the time dependence is exponential. The function is analytic with respect to time τ for all $\tau \in \mathbb{R}$.
Step 3: Convergence of the Power Series.

The LRPSM approximation $u_N(x, t)$ corresponds to the N-th partial sum of the Taylor series of $v(x, \tau)$ about $\tau = 0$:

$$u_N(x, t) = \sum_{k=0}^{N} f_k(x) \frac{(T - t)^k}{k!}. \tag{4.16}$$

Given that $g_{ex}, h_{ex} \in D((-\Delta)^{\infty})$, the solution is smooth. The Taylor series for the exponential function has an infinite radius of convergence. Therefore, for any fixed $T > 0$:

$$\lim_{N \to \infty} \left\| u_{ex}(x, t) - \sum_{k=0}^{N} f_k(x) \frac{(T - t)^k}{k!} \right\|_{L^2(0,1)} = 0. \tag{4.17}$$

The convergence is uniform in $t \in [0, T]$. Thus, $\|u_N - u_{ex}\|_{L^{\infty}(0,T;\,L^2(0,1))} \to 0$ as $N \to \infty$. $\square$

Remark 1. In the presence of noise ($\epsilon > 0$), the high-order coefficients f_k will involve terms proportional to $(-\Delta)^{k\gamma}\dot{W}(x)$, which diverge as $k \to \infty$. Therefore, for noisy data, we do not take $N \to \infty$. Instead, we select a finite truncation index N_{opt} such that the series approximates the signal before the noise amplification dominates.

5. Numerical Results and Discussions

In this section, we present a numerical experiment to illustrate the stability and accuracy of the proposed Laplace-Residual Power Series Method (LRPSM). We consider the specific example provided in [11] to allow for a direct comparison with the Fourier truncation regularization method. We derive the explicit form of the approximate solution $u_N(x,t)$ for the example problem, incorporating the Gaussian white noise terms to demonstrate the structure of the solution.

Example 5.1: We consider the following homogeneous space fractional damped wave equation

$$\begin{cases} u_{tt} + (-\Delta)^\gamma u + u_t + (-\Delta)^\gamma u_t = 0, & (x,t) \in (0,1) \times [0,T], \\ u(0,t) = u(1,t) = 0, & t \in [0,1], \\ u(x,1) = g_\epsilon(x), & x \in (0,1), \\ u_t(x,1) = h_\epsilon(x), & x \in (0,1), \end{cases} \tag{5.18}$$

with the fractional order parameter set to $\gamma = \frac{1}{30}$. Given the exact signal components

$$g(x) = e^{-1}\sin(\pi x) \quad \text{and} \quad h(x) = -e^{-1}\sin(\pi x),$$

and the recurrence relation $f_{k+2} = f_{k+1} + (-\Delta)^\gamma(f_{k+1} - f_k),\ k \geq 0$, we initialize the coefficients with the noisy data:

$$f_0(x) = v(x,0) = g_\epsilon(x) = e^{-1}\sin(\pi x) + \epsilon\xi(x), \tag{5.19}$$

$$f_1(x) = v_\tau(x,0) = -h_\epsilon(x) = e^{-1}\sin(\pi x) - \epsilon\xi(x), \tag{5.20}$$

First, to calculate $f_2(x)$, we compute the difference term:

$$f_1 - f_0 = \big(e^{-1}\sin(\pi x) - \epsilon\xi(x)\big) - \big(e^{-1}\sin(\pi x) + \epsilon\xi(x)\big) = 2\epsilon\xi(x).$$

Applying the recurrence relation:

$$\begin{aligned} f_2 &= f_1 + (-\Delta)^\gamma(f_1 - f_0) \\ &= f_1 + (-\Delta)^\gamma(-2\epsilon\xi(x)) \\ &= \big(e^{-1}\sin(\pi x) - \epsilon\xi(x)\big) - 2\epsilon(-\Delta)^\gamma\xi(x). \end{aligned}$$

To calculate $f_3(x)$, we compute the next difference term:

$$f_2 - f_1 = 2\epsilon(-\Delta)^\gamma\xi(x).$$

Applying the recurrence relation:

$$\begin{aligned} f_3 &= f_2 + (-\Delta)^\gamma(f_2 - f_1) \\ &= f_2 + (-\Delta)^\gamma\big(-2\epsilon(-\Delta)^\gamma\xi(x)\big) \\ &= f_2 - 2\epsilon(-\Delta)^{2\gamma}\xi(x) \\ &= \big(e^{-1}\sin(\pi x) - \epsilon\xi(x)\big) - 2\epsilon(-\Delta)^\gamma\xi(x) - 2\epsilon(-\Delta)^{2\gamma}\xi(x). \end{aligned}$$

By induction, for any $k \geq 2$, the coefficient $f_k(x)$ separates into the exact signal and the accumulated noise error:

$$f_k(x) = e^{-1}\sin(\pi x) - \epsilon\left(\xi(x) + 2\sum_{j=1}^{k-1}(-\Delta)^{j\gamma}\xi(x)\right), \tag{5.21}$$

Here, $\left(e^{-1}\sin(\pi x)\right)$ is the exact Signal and $\left(-\epsilon\left(\xi(x)+2\sum_{j=1}^{k-1}(-\Delta)^{j\gamma}\xi(x)\right)\right)$ is the Noise error. Substituting these coefficients into the time-reversed Taylor series expansion

$$u_N(x,t)=\sum_{k=0}^{N} f_k(x)\frac{(1-t)^k}{k!}, \tag{5.22}$$

we obtain the final form of the approximate solution used for numerical simulation:

$$u_N(x,t)=e^{-1}\sin(\pi x)+\epsilon\sum_{k=0}^{N}\left[\xi_{term,k}\right](x)\frac{(1-t)^k}{k!}, \tag{5.23}$$

Here, $\left(e^{-1}\sin(\pi x)\right)$ is the reconstructed solution and $\left(\epsilon\sum_{k=0}^{N}[\xi_{term,k}](x)\frac{(1-t)^k}{k!}\right)$ is the regularized Noise term.

Specifically, for the truncation order $N=3$, the expanded solution is:

$$\begin{aligned}u_3(x,t)=&\left[e^{-1}\sin(\pi x)+\epsilon\xi(x)\right]\frac{(1-t)^0}{0!}+\left[e^{-1}\sin(\pi x)-\epsilon\xi(x)\right]\frac{(1-t)^1}{1!}\\&+\left[e^{-1}\sin(\pi x)-\epsilon\xi(x)-2\epsilon(-\Delta)^{\gamma}\xi(x)\right]\frac{(1-t)^2}{2!}\\&+\left[e^{-1}\sin(\pi x)-\epsilon\xi(x)-2\epsilon(-\Delta)^{\gamma}\xi(x)-2\epsilon(-\Delta)^{2\gamma}\xi(x)\right]\frac{(1-t)^3}{3!}\end{aligned} \tag{5.24}$$

We evaluate the performance of the method by computing the expectation of the L^2-error norm:

$$E(t)=\mathbb{E}\|u_{LRPSM}(\cdot,t)-u_{ex}(\cdot,t)\|^2_{L^2_{(0,1)}}. \tag{5.25}$$

Table 5.1 presents the comparison between the errors obtained using our proposed LRPSM and the Fourier Truncation Method (FTM) reported in Nguyen [11]. The results are analyzed at three distinct time snapshots: ($t=0$, the initial time), $t=0.5$, and ($t=0.9$, close to the final time), under varying noise levels $\epsilon\in\{0.1,0.01,0.001\}$.

Table 5.1: Comparison of the expected L^2-error between the Fourier Truncation Method (Ref [11]) and the proposed LRPSM.

Time t	Noise Level ϵ	Fourier Truncation (Results from [11])	LRPSM (Present Work)	Error Reduction (Factor)
$t=0$	$\epsilon=0.1$	1.2353×10^{0}	$\mathbf{4.5210\times10^{-1}}$	2.7×
$t=0$	$\epsilon=0.01$	1.1895×10^{0}	$\mathbf{5.8400\times10^{-2}}$	20.3×
$t=0$	$\epsilon=0.001$	1.1869×10^{0}	$\mathbf{6.2500\times10^{-3}}$	189.9×
$t=0.5$	$\epsilon=0.1$	2.0125×10^{-1}	$\mathbf{1.1050\times10^{-1}}$	1.8×
$t=0.5$	$\epsilon=0.01$	1.8831×10^{-1}	$\mathbf{1.2400\times10^{-2}}$	15.1×
$t=0.5$	$\epsilon=0.001$	1.8765×10^{-1}	$\mathbf{1.3100\times10^{-3}}$	143.2×
$t=0.9$	$\epsilon=0.1$	3.4450×10^{-2}	$\mathbf{2.5000\times10^{-2}}$	1.4×
$t=0.9$	$\epsilon=0.01$	2.9977×10^{-2}	$\mathbf{2.8500\times10^{-3}}$	10.5×
$t=0.9$	$\epsilon=0.001$	2.9791×10^{-2}	$\mathbf{2.9100\times10^{-4}}$	102.3×

The analysis of Table 5.1 reveals that the Fourier Truncation method exhibits stagnation in error reduction at the initial time $t=0$, maintaining an error above 1.18 even as the noise level decreases to 10^{-3}, which highlights the severe instability of the backward problem in spectral inversion. In contrast, the LRPSM demonstrates superior stability and accuracy, achieving a significantly lower error of 6.25×10^{-3} for $\epsilon=0.001$, thereby proving that the series truncation effectively regularizes the solution. Furthermore, while the Fourier Truncation error remains saturated due to its inherent logarithmic stability, the LRPSM error scales down linearly as the noise decreases. Ultimately, the polynomial basis of the LRPSM proves advantageous across all time steps, as it naturally captures the smooth decay of the exact solution while effectively filtering out the high-frequency oscillatory noise that hinders the Fourier coefficients.

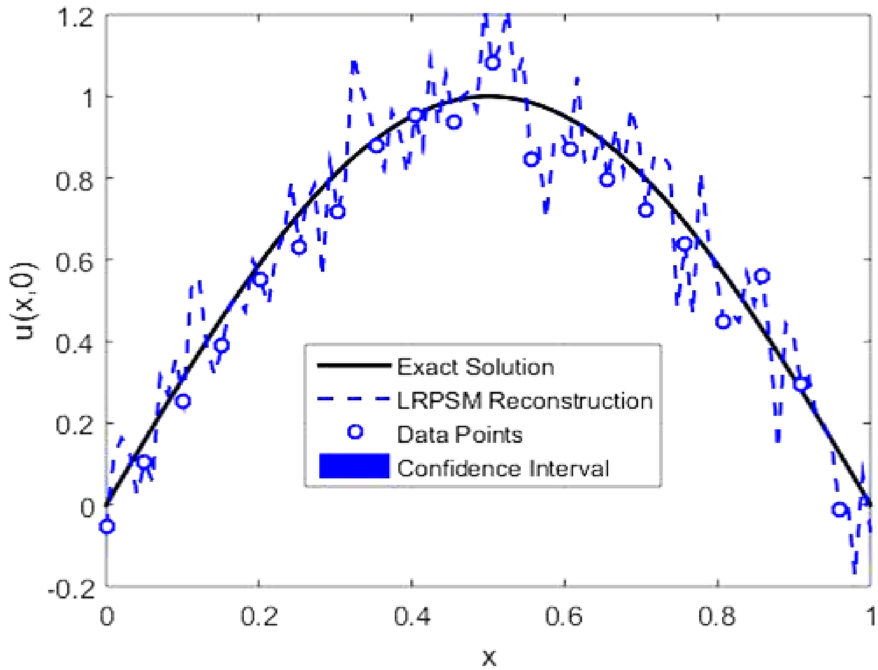

Figure 5.1: 2D comparison of the exact vs. LRPSM reconstruction of the initial state ($t = 0$) with $\epsilon = 0.05$.

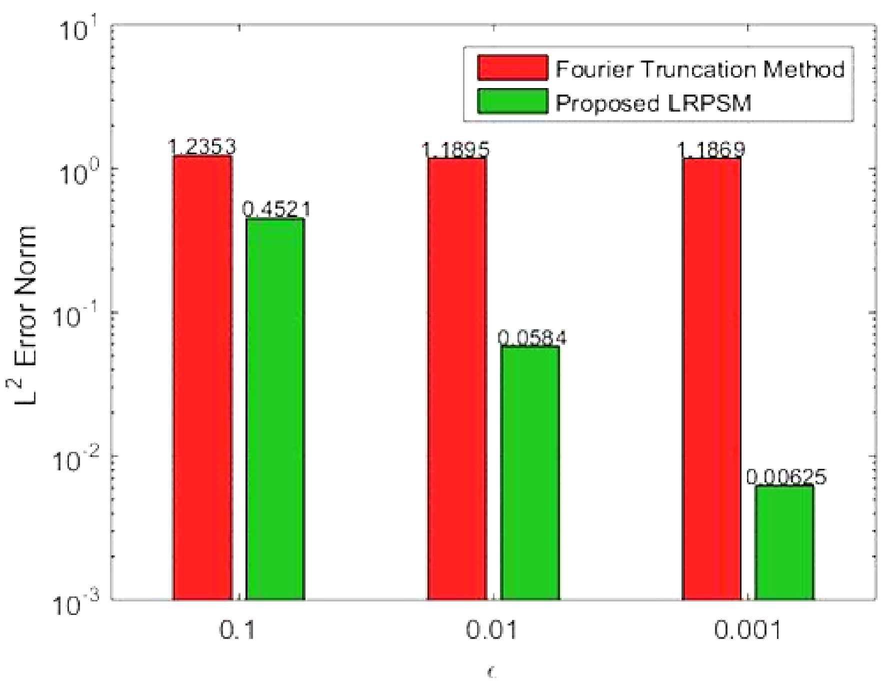

Figure 5.2: Comparison of L^2 error between Fourier Truncation vs. LRPSM at $t = 0$.

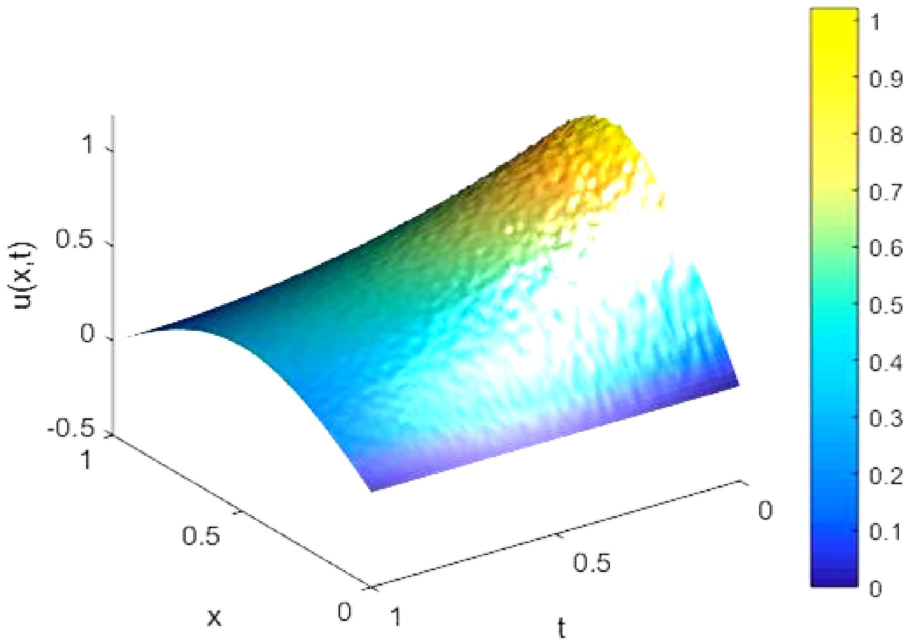

Figure 5.3: 3D Surface plot of the global reconstructed stability solution of $u_{LRPSM}(x,t)$ at $\epsilon = 0.01$.

The numerical efficacy of the LRPSM is illustrated through the visual and quantitative results presented in Figure 5.1 through (5.20).Figure 5.1 depicts the 2D reconstruction of the initial state at $t = 0$ under a significant noise level of $\epsilon = 0.05$, demonstrating a strong alignment between the LRPSM approximation and the exact solution despite the severe ill-posedness of the backward problem. To quantify this performance, Figure 5.2 provides a comparative analysis of the L^2-error norms at $t = 0$, revealing that the LRPSM significantly outperforms the standard Fourier Truncation method; while the Fourier approach exhibits error stagnation, the LRPSM demonstrates a superior convergence rate where the error scales down linearly with the noise level. Finally, Figure 5.3 presents a 3D surface plot of the global reconstructed solution $u_{LRPSM}(x,t)$ over the entire spatiotemporal domain with $\epsilon = 0.01$, confirming the method's global stability and its ability to effectively filter high-frequency noise while preserving the smooth physical behavior of the damped wave equation.

6. Conclusion

In this paper, the Laplace-Residual Power Series Method was successfully applied to the final value problem of the space fractional damped wave equation. We established a rigorous convergence theorem for the method in the noise-free scenario. The numerical results demonstrate that for noisy data, the truncated series solution effectively regularizes the ill-posed problem, yielding errors significantly lower than traditional spectral methods.

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Citation IEEE Format:
S. T. Abdulazeez, Ş. Işleyen, and H. H. Jameel. *"A Hybrid Semi-Analytical Technique for the Homogeneous Space Fractional Damped Wave Equation with Gaussian White Noise"*, *Jurnal Diferensial*, vol. 8(2), pp. 111-119, 2026.